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WELDING RESEARCH

29, PARK CRESCENT, LONDON, W.1

ASSOCIATION NOTES

ANNUAL GENERAL MEETING, 1953

The Eighth Annual General Meeting of the Association will be held at the Engineering Research Station, Abington, near Cambridge, on 16th July, 1953, at 12 noon, followed by luncheon and an exhibition of researches in progress.

WELDING RESEARCH PRIZE BY BRITISH OXYGEN COMPANY

We are glad to announce that one of the winning papers for the final year of this competition, submitted by Dr. D. C. Moore, will be published in the next issue of *Welding Research*.

Dr. Moore was the joint winner of the prize with Mr. E. G. Hinds and Mr. M. D. Jackson. The titles of the two papers were as follows:

"Effect of Impurities in Argon for Argon-Arc Welding of Aluminium and its Alloys," by D. C. Moore, B.Met., M.Sc., Ph.D.

"The Development of Improved Weldability Tests for Steels," by E. G. Hinds, B.A., and M. D. Jackson, B.Sc.

WELDING CONGRESS IN HAMBURG

The Deutscher Verband für Schweisstechnik EV. have recently announced that they will be holding a Welding Congress in Hamburg from 15th-19th June, 1953.

The first two days will be devoted to the affairs of the Association, including committee meetings and Annual Council meeting, but from the 17th to the 19th June there will be a series of lectures open to all welding engineers and others interested in welding. The main emphasis of the Congress is on welding in shipbuilding, including lectures on flame cutting, light-alloy welding, etc., but other aspects of the welding industry will also be included, such as ultrasonic flaw detection, relief and control of residual stresses by gas heating, etc.

A full programme is included in the February, 1953, issue of Schweissen und Schneiden. For further details and application for formal invitation, please write to the above Association at the following address: Düsseldorf, Harkortstrasse 27, Postschliesstach 3042.

SOME ALUMINIUM-ZINC-MAGNESIUM ALLOYS—AN EXAMINATION OF THEIR TENSILE PROPERTIES AND OXY-ACETYLENE WELDING CHARACTERISTICS

An examination has been made of the tensile properties and oxy-acetylene welding characteristics of a number of aluminium-zinc-magnesium alloys. The alloys examined were susceptible to heat treatment and should prove of value for applications where high strength and moderately good welding properties are required. The results obtained in gas welding tests made under restraint suggest that the presence of copper in aluminium-zinc-magnesium alloys markedly impairs their resistance to cracking during welding, and it would seem desirable to maintain the copper content as low as possible in aluminium-zinc-magnesium alloys to be fabricated by welding.
The Association is indebted to The Aluminium Development Association for permission to publish this work.

By W. I. PUMPHREY,* M.Sc., Ph.D.

INTRODUCTION

In earlier papers, Pumphrey and Jennings[1] and Pumphrey and Moore[2] considered the fundamental factors determining the incidence of cracking at temperatures above and below the solidus during the welding of aluminium alloys. Moore[3] later employed the theory of Pumphrey and Jennings to show how the incidence of cracking at temperatures above the solidus may be reduced for many of the high strength aluminium alloys.

The present contribution is concerned with the development of aluminium alloys of inherently high strength and good welding properties.

The essential requirements for a good "welding alloy" are:

(*a*) The alloy must not be subject to cracking at high temperatures during the welding operation, nor at lower temperatures during the subsequent cooling of the weld complex.

(*b*) The alloy must be easy to weld and must not have such an extended solidification range that the weld metal becomes "sluggish" or the weld bead tends to "fall through" during the welding operation.

(*c*) The alloy should have no tendency to superficial blistering or internal porosity in the vicinity of the weld bead.

(*d*) The alloy should have good mechanical properties both before and after welding, and if welded in the heat-treated condition should not suffer any serious deterioration in properties on exposure to the heat of the welding arc or flame for a short period.

(*e*) The alloy should be reasonably resistant to all forms of corrosion both before and after welding.

(*f*) The alloy should have good casting and rolling properties and should not contain such costly alloying elements that its selling price is prohibitive.

On the basis of the results obtained in the investigations described in previous papers a selection was made of a number of alloys giving little or no cracking in ring casting and restrained welding tests and their tensile and general welding properties were examined. Particular attention was given to alloys in the aluminium-zinc-magnesium system and the observations made on selected alloys in this system are recorded and discussed in the present paper.

PREVIOUS WORK

It has been known for a number of years that aluminium alloys containing zinc and magnesium as the principal alloying elements and with or without additions of copper, have good mechanical properties, and there is an increasing number of specifications—for example, DTD 683, DTD 687 and DTD 693—covering the use of such alloys in various forms.

In general, the mechanical properties obtainable with aluminium-zinc-magnesium alloys are higher than those of fully heat-treated duralumin-type alloys, and it has been shown[4, 5, 6] that such alloys may have an elastic limit of between 25 and 32 tons per sq.in. in the rolled condition, and between 35 and 38 tons per sq.in. in the extruded condition, with an ultimate tensile strength of up to 42 tons per sq.in. Alloys of this type may also possess a fatigue limit as high as 11 tons per sq.in. as compared with a maximum fatigue limit of 9 tons per sq.in. for any other light alloy.

A number of investigators have observed that aluminium-zinc-magnesium alloys are less easy to roll than many other light alloys, that they have a lower capacity for plastic deformation before breakage, and that they are difficult to cold work since ageing occurs very rapidly after quenching. However, Mott and Thompson[7]—who investigated the casting, fabricating and mechanical properties of alloys free from copper and containing 3 to 12 per cent. zinc, 1 to 5 per cent. magnesium and 1 to 2 per cent. manganese—

* Research Manager, Murex Welding Processes Ltd. Formerly Investigator in charge of Welding Research on behalf of the Aluminium Development Association at the University of Birmingham.

have reported that the majority of the alloys within this range of compositions have good hot-working properties and appear to roll and forge easily. A systematic investigation of the casting and working properties of aluminium-zinc-magnesium alloys has also been made by Cook, Chadwick and Muir.[8]

Aluminium-zinc-magnesium alloys are reported to have a high resistance to corrosion by sea-water,[6] but may be subject to stress corrosion, particularly in the cold-aged condition.[9, 10, 11] Considerable attention has been paid to the effect of heat treatment and of additions of other alloying elements on the stress corrosion properties, and it has been suggested that corrosion of this type may be overcome by annealing[4, 10, 12] or by the addition of small quantities of such elements as manganese,[13, 14] chromium,[7, 15, 16] or vanadium.[7, 15, 16]

Although aluminium-zinc-magnesium alloys may be forged at temperatures of the order of 400 deg. C. and spot welded electrically, they have often been reported as unsuitable for brazing or oxy-acetylene welding, and Pendleton[17] observed a pronounced tendency to cracking in welds made under restraint in an alloy containing approximately 5·5 per cent. zinc, 2·8 per cent. magnesium and 0·5 per cent. copper. The majority of investigations into the properties of alloys of this type, however, have been carried out with alloys containing from 0·25 to 2 per cent. of copper as well as smaller quantities of other elements such as chromium, silicon and manganese, and no examination of the welding properties of alloys free from copper appears to have been reported. In published statements of the properties of aluminium-zinc-magnesium alloys free from copper, no adequate consideration has been made of the welding properties of the alloys in question.

EXPERIMENTAL WORK

An examination was made of selected aluminium-zinc-magnesium alloys, produced from virgin metals of commercial purity, and cast and rolled under industrial conditions, as well as of alloys made to a number of commercial specifications. Restrained weld tests, using oxy-acetylene welding and filler rods of the same composition as the sheets being welded were carried out on all the alloys selected for investigation and an examination was made of the tensile properties of the alloys before welding and of the tensile properties of welds made in heat-treated and annealed material.

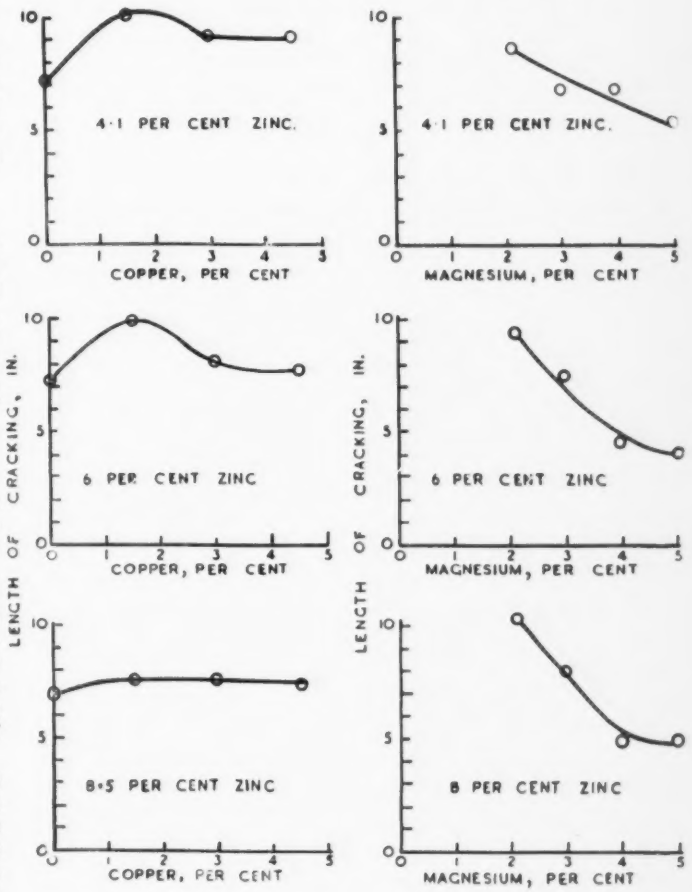
The welding and mechanical tests were on material rolled down to the form of sheets 0·080 in. in thickness, the tensile test pieces from the unwelded material being machined to conform to B.S.485 with the parallel portion of each specimen transverse to the direction of rolling of the sheet. Tensile tests of the alloys after welding were carried out on test pieces machined from those welded sheets in which weld cracking was not so severe as to render testing impossible, the weld being located at the centre of the parallel portion of the test piece and transverse to the length. Before tensile testing, the excess weld metal in the reinforcement and underbead was machined away to give specimens of uniform thickness along their length.

Four tensile tests were carried out on every alloy in each condition, that is, as annealed and heat-treated and as welded in both these conditions. The alloys were tensile tested at a rate of straining of 0·3 in. per minute and examined for ultimate tensile strength, 0·1 per cent. proof stress (in the case of the unwelded specimens only), elongation (per cent. on 2 in.), and, in the case of the welded specimens, position of fracture. Each test piece was tested for hardness before mechanical testing, the hardnesses recorded in the tables in this paper being the averages of at least three hardness impressions taken at the ends of each specimen.

As a preliminary examination of the welding properties of aluminium-zinc-magnesium alloys, an investigation was made of the effect of variation in the zinc, magnesium and copper contents on the tendency for cracking to occur during the welding of alloys with compositions based on DTD. 363A and DTD. 687. The tendency to cracking was assessed by means of ring casting tests.[1, 2]

Two basic alloys of commercial purity were used for the investigation, additions being made to these two alloys to vary the percentages of zinc, magnesium and copper as required. The two basic alloys were of the compositions given in Table I.

The first alloy was used to determine the effect on cracking of variation in the zinc and copper contents between 4·1 and 8·5 per cent. and 0 and 4·5 per cent. respectively at a constant magnesium content of 2·93 per cent.; the second alloy was used to determine the effect of variation in the zinc and magnesium contents between 4·1 and 8·5 per cent. and 2·08 and 5·0 per cent. respectively at a fixed copper content of 0·1 per cent. The results of the ring casting tests are reproduced in Fig. 1. The amount of cracking in alloys derived from the first basic alloy was found to increase slightly with increase in the copper content to 1·5 per cent., and then to decrease to the original value with further



(a) Alloys based on first basic alloy—2·93 per cent. Mg Content.

(b) Alloys based on second basic alloy—0·10 per cent. Cu Content.

Fig. 1. The effect of variation in the zinc, magnesium and copper contents on the cracking of ring castings in alloys based on specifications DTD 363A and DTD 687.

Table I.—Compositions of Two Basic Alloys used in Preliminary Investigations of Welding Properties of Aluminium-Zinc-Magnesium Alloys

Alloy Reference No.	Copper (per cent.)	Magnesium (per cent.)	Manganese (per cent.)	Zinc (per cent.)	Chromium (per cent.)	Silicon (per cent.)	Iron (per cent.)
1	—	2·93	0·46	4·10	—	0·12	0·23
2	0·10	2·08	0·10	4·10	0·25	0·12	0·22

additions of copper; this effect was less marked at the higher percentages of zinc. The minimum length of cracking (7 in.) which was obtained with the basic alloy containing no copper was, however, high. The effect of variation in the magnesium content on the tendency to cracking of alloys derived from the second basic alloy was more marked, the total length of cracking in the alloy containing 6 per cent. zinc decreasing from 9·4 in. to 4·1 in. with increase in the magnesium content from 2·08 to 5·0 per cent. Again, however, the minimum length of cracking obtained was high.

From the results of the ring casting tests it would be expected that alloys based on specifications DTD 363A and DTD 687 would, in general, have poor welding properties, and that no marked improvement in the welding properties would be obtained by varying the amount of zinc or copper in the alloys. Weld cracking would, however, be expected to decrease with increase in the magnesium content. The general level of cracking in the alloys free from copper was lower than that in the alloys containing copper, and it was thought to be of interest to examine the welding and mechanical properties of aluminium-zinc-magnesium alloys of commercial purity but free from copper. For this work eight alloys were selected containing varying amounts of zinc and magnesium and free from copper, manganese and chromium.

Table II.—Composition of Aluminium-Zinc-Magnesium Alloys

Cast	Nominal Composition		Actual Composition			
	Zn (per cent.)	Mg (per cent.)	Zn (per cent.)	Mg (per cent.)	Fe (per cent.)	Si (per cent.)
A	4	2	4·31	2·05	0·21	0·12
B	4	2	4·31	2·05	—	—
A	4	4	3·78	4·22	—	—
B	4	4	3·82	4·02	—	—
A	4	5	4·10	5·42	0·22	0·12
A	6	2	5·86	2·08	—	—
A	6	4	5·86	4·26	0·21	0·12
A	6	5	6·19	5·36	—	—
B	6	5	5·98	5·21	—	—
A	8	2	8·51	2·08	0·20	0·09
B	8	2	7·77	2·30	—	—
A	8	4	8·51	4·15	—	—

The chemical analyses of the alloys examined are recorded in Table II. Certain of the alloys, in particular those containing 6 per cent. zinc with 5 per cent. magnesium and 8 per cent. zinc with 2 per cent. magnesium were extremely brittle, and considerable difficulty was experienced in rolling them to sheet form. In order to obtain a sufficient quantity of finished material a second cast was made of certain of the alloys. The chemical analyses of both casts, when two casts of any particular alloy were made, are recorded separately in Table II, and the tensile and welding properties of the material from the two casts are also recorded separately in Tables III and IV. Four of the alloys were analysed for iron and silicon and all four were found to contain approximately 0·2 per cent. iron and 0·1 per cent. silicon. Since all the ingots were made from virgin materials of the same commercial purity, it may be assumed that the iron and silicon contents of the remaining alloys were of the same order. The alloys were found to contain no appreciable quantities of impurities other than iron and silicon.

The preliminary hot rolling of the alloys was at 330 deg. C., rolling at higher temperatures being found to cause "alligating" of certain of the cast slabs. Edge cracking also occurred to a varying extent in each alloy. After shearing, the hot rolled blanks were finally cold rolled to the finishing thickness of 0·080 in., reductions of the order of 40 to 50 per cent. being given to most of the alloys during cold rolling. Intermediate and final annealing was at 360 deg. C., and afterwards the sheets were flattened by one pass through a roller leveller.

After annealing and flattening, the sheets were sectioned: one part of each sheet being reserved for the examination

Table III.—Mechanical Properties of Aluminium-Zinc-Magnesium Alloys

Cast	Nominal Composition		Results of Tests on Annealed Material				Results of Tests on Heat-treated Material			
	Zn (per cent.)	Mg (per cent.)	U.T.S. (tons per sq.in.)	0·1 per cent. proof stress (tons per sq.in.)	Elongation* (per cent.)	Hardness (D.P.H.)	U.T.S. (tons per sq.in.)	0·1 per cent. proof stress (tons per sq.in.)	Elongation* (per cent.)	Hardness (D.P.H.)
A	4	2	15·6	5·2	20·5	69	22·8	16·9	10·4	127
B	4	2	13·4	4·8	8·7	67	19·7	15·0	7·5†	119
A	4	4	15·3	4·1	18·4	67	26·3	22·5	3·5†	171
B	4	4	14·8	3·9	16·3	59	25·6	23·6	4·0†	144
A	4	5	14·0	3·8	7·3	72	28·2	—	2·0	183
A	6	2	12·9	3·5	18·1	55	30·6	26·1	4·1†	163
A	6	4	14·5	3·2	16·4	64	34·6	30·8	2·3†	189
A	6	5	15·6	3·8	12·75	74	27·5	26·6	1·8†	196
B	6	5	13·9	3·9	11·75	65	30·7	28·9	1·1†	205
A	8	2	12·5	3·3	19·8	54	30·8	27·0	1·9†	181
B	8	2	12·6	3·6	16·75	56	32·1	29·15	1·7†	179
A	8	4	13·4	3·5	15·0	58	18·0	7·1	7·8	98

* Percentage elongation on 2 in. unless otherwise stated. † Percentage elongation on 1·5 in.

Table IV.—Mechanical Properties of Welds in Aluminium-Zinc-Magnesium Alloys

Cast	Nominal Composition		Results of Tests on Welds in Annealed Material				Results of Tests on Welds in Heat-treated Material			
	Zn (per cent.)	Mg (per cent.)	Average length of Cracking in Restrained Weld Test (in.)	U.T.S. (tons per sq.in.)	Elongation* (per cent.)	Position of Fracture	Average Length of Cracking in Restrained Weld Test (in.)	U.T.S. (tons per sq.in.)	Elongation* (per cent.)	Position of Fracture
A	4	2	2·1	13·7	4·4	In weld or at edge of weld	1·9	17·4	9·0	In weld or in basis metal
B	4	2	2·8	—	—	At edge of weld	1·6	14·5	6·0	In weld or at edge of weld
A	4	4	0·8	14·7	4·1	In weld or in basis metal†	1·0	15·55	3·5	In weld
B	4	4	0·8	15·0	4·25	In weld or in basis metal	1·1	13·9	2·6	In basis metal
A	4	5	0·4	15·5	3·7	In weld or at edge of weld	1·1	16·0	2·5	In weld
A	6	2	2·0	14·4	2·5	At edge of weld	2·6	15·2	4·3	In basis metal
A	6	4	0·4	15·0	5·5	In basis metal	2·2	18·5	2·5	In weld
A	6	5	1·5	15·2	3·3	In basis metal	1·2	14·6	1·25	In weld
B	6	5	0·2	14·9	4·75	In basis metal	0·5	14·9	1·5	In weld or at edge of weld
A	8	2	2·2	14·8	1·5	In weld	5·1	14·3	1·5	In weld
B	8	2	1·6	14·8	4·0	In basis metal	3·2	14·2	4·5	In basis metal
A	8	4	1·0	15·1	5·0	In basis metal	0·9	13·2	1·75	In weld or at edge of weld

* Percentage elongation on 2 in.

† In basis metal indicates that the fracture occurred at a point remote from the heat affected zone of the weld.

of the properties of the material in the annealed condition and the other part being heat-treated for the examination of the properties in the fully heat-treated condition. The eight alloys were given a standard heat treatment consisting of a solution treatment at 450 deg. C., followed by ageing for eighteen hours at 135 deg. C.

TENSILE PROPERTIES OF THE ALLOYS

The results of hardness and tensile tests of the alloys in the heat-treated and annealed conditions are summarised in Table III, and the results of tensile tests of welds made in the alloy in these two conditions in Table IV.

There are a number of slightly anomalous results in the two tables, but most of these may be ascribed to irregularities in the surface quality of the specimens: in the heat-treated condition and after welding, the surfaces of many of the sheets tended to be loose and flaky, and the degree of looseness varied in a somewhat irregular fashion across the surfaces of the specimens.

Table III indicates that all the alloys examined were particularly susceptible to heat treatment, although in the heat-treated condition the elongation of the majority of the alloys was low. The strengths of the heat-treated alloys were markedly reduced by welding, the strengths of the welded alloys being only some 50 to 60 per cent. of their strengths before welding. Even with this reduction, however, the ultimate tensile strengths of all the welded alloys examined were in excess of 13 tons per sq.in. The elongations of the heat-treated alloys were very similar before and after welding.

After welding, the tensile properties of the annealed alloys were similar to those obtained before welding, although the elongations were much lower.

As indicated in Table IV, the location of fracture in the tensile tests of the welded alloys varied in a somewhat irregular fashion with alloy content.

The tensile properties of the alloys examined by Pendleton,[17] which contained approximately 5·5 per cent. zinc and 2·8 per cent. magnesium, were, in general, higher than those of an alloy of a similar composition tested in the present investigation, although the elongations after welding were lower. The alloys examined by Pendleton, however, contained between 0·48 and 0·72 per cent. copper, and his tensile figures are, therefore, not strictly comparable with those obtained in the present work.

WELDING PROPERTIES OF THE ALLOYS

The tendency of all the alloys to cracking in the restrained weld test was considerably lower than that of the alloys to DTD 363A and 687 subjected to ring casting tests—as shown by a comparison of the results in Fig. 1 with those in Table IV. In only one alloy, namely, that containing 8 per cent. zinc and 2 per cent. magnesium welded in the heat-treated condition, was there any tendency to cracking along the entire length of the weld. As with the alloys to DTD 363A and 687 there was, in general, a decrease in weld cracking at any one zinc content with increase in the magnesium content. Increase in the zinc content at any one magnesium content had little or no effect on the tendency to cracking. The susceptibility to cracking during welding was similar in both the heat-treated and annealed materials, although there was some slight tendency for the alloys welded in the heat-treated condition to crack to a greater extent than the corresponding alloys welded in the annealed condition. There was some tendency for cracking to occur below the solidus, at temperatures of the order of 200 deg. C., in certain of the alloys having low elongation, the more so in the alloys welded in the heat-treated condition; but the general tendency toward such cracking was slight. Cracking in the welds occurred somewhat indiscriminately at the centres and at the edges of the welds and occasionally at both places at once, as shown in Fig. 2. The location of cracking in the welds appeared to be independent of the composition

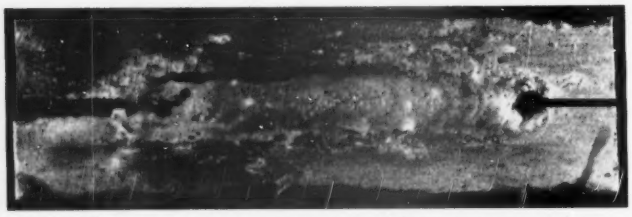
of the material, and of its condition—whether heat-treated or annealed—before welding.

With a number of the alloys there was pronounced superficial blistering of the basis metal adjacent to the weld bead, and a similar effect was noticed by Pendleton.[17] The degree of blistering appeared to be independent of the zinc content and of the structural condition of the material before welding, but appeared to increase with increasing magnesium content at any one zinc content, as shown by Fig. 3. Microscopic examination indicated that the degree of internal porosity was closely associated with that of surface blistering. In the alloys containing weld porosity the porosity was concentrated in a narrow band on either side of the weld bead. Severe surface blistering makes it extremely difficult for the operator to deposit a satisfactory run of weld metal, and it would seem that at any one zinc content welding would be difficult at a magnesium content exceeding 5 per cent.

In addition to their tendency to blister during welding, the surfaces of the rolled sheets of high total alloy content tended to be loose and flaky in the heat-treated (and occasionally in the annealed) condition before welding: this was less pronounced in the alloys of lower total alloy content. It should be borne in mind, however, that this observation was made with alloys cast and rolled on a small scale under laboratory conditions, and these findings may not apply to alloys of the same compositions produced on a large scale under works conditions.

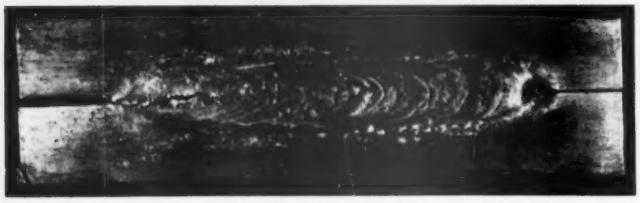
EXAMINATION OF ALUMINIUM-ZINC-MAGNESIUM ALLOYS CONTAINING COPPER

The results recorded in Table IV confirm the suggestion made earlier that the welding properties of the copper-free aluminium-zinc-magnesium alloys are superior to those of the alloys containing copper. In order to examine further the effect of copper on the welding and mechanical properties of aluminium-zinc-magnesium alloys, five alloys of commercial purity were obtained, with the kind co-operation of Member Companies of the Aluminium Development Association, containing approximately 5·5 per cent. zinc and 2·5 per cent. magnesium and with copper contents of 0·44 to 2·04 per cent. The analyses of these alloys are recorded in Table V.



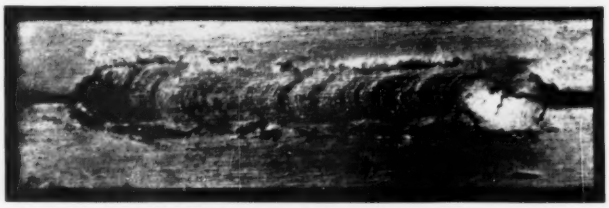
(a) Alloy containing 8 per cent. zinc and 2 per cent. magnesium welded in heat-treated condition.

(b) Alloy containing 6 per cent. zinc and 2 per cent. magnesium welded in annealed condition.

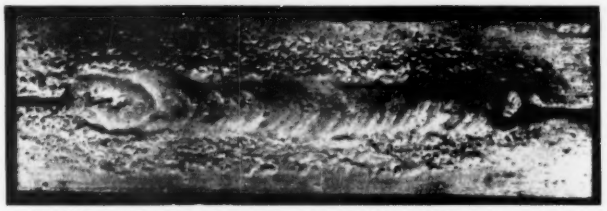


(c) Alloy containing 8 per cent. zinc and 2 per cent. magnesium welded in annealed condition.

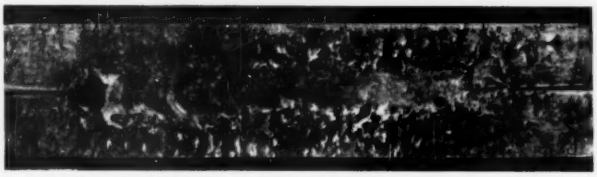
Fig. 2. Cracking in welds in some aluminium-zinc-magnesium alloys free from copper. Unetched. ×1.



(a) Alloy containing 6 per cent. zinc and 2 per cent. magnesium welded in heat-treated condition.



(b) Alloy containing 6 per cent. zinc and 4 per cent. magnesium welded in heat-treated condition.



(c) Alloy containing 6 per cent. zinc and 5 per cent. magnesium welded in heat-treated condition.

Fig. 3. Blistering in weld area in some aluminium-zinc-magnesium alloys free from copper. Unetched. ×1.

TENSILE PROPERTIES OF THE ALLOYS

The results of tensile tests of the five alloys listed in Table V in the heat-treated and annealed conditions are recorded in Table VI, and the results of tensile tests on welds made in the alloys in these two conditions are summarised in Table VII. The strengths of the heat-treated alloys were considerably reduced by welding and the properties of the five alloys after welding were of the same order as those of the aluminium-zinc-magnesium alloy of approximately the same zinc and magnesium content but free from copper in the corresponding conditions (Table IV).

WELDING PROPERTIES OF THE ALLOYS

Restrained weld tests were carried out on all the alloys, the welding being done by two operators working alternately using the same welding technique and parent metal filler rods. The results obtained by the two operators with any one alloy were practically identical and the cracking figures obtained by the two operators have been averaged and are so recorded in Tables VIII and IX.

Four of the five alloys examined were found to be suscep-

Table V.—Compositions of Aluminium-Zinc-Magnesium Alloys of Commercial Purity containing 0·44 to 2·04 per cent. of Copper

Alloy Reference No.	Specification	Chemical Composition							
		Zn (per cent.)	Mg (per cent.)	Cu (per cent.)	Fe (per cent.)	Si (per cent.)	Mn (per cent.)	Ti (per cent.)	Cr (per cent.)
1	DTD 687	5·40	2·20	0·44	0·29	0·12	0·50	0·02	0·01
2	DTD 687	5·90	2·68	0·98	0·50	0·34	0·30	—	0·14
3	Rex 124	5·35	2·58	1·09	0·26	0·08	0·27	0·07	0·08
4	DTD 687	5·30	2·55	1·32	0·25	0·13	0·26	0·08	0·08
5	DTD 363	5·05	2·58	2·04	0·38	0·19	0·50	0·04	Nil

Table VI.—Mechanical Properties of Alloys Listed in Table V

Alloy	Results of Tests on Annealed Material			Results of Tests on Heat Treated Material		
	U.T.S. (tons per sq.in.)	0·1 per cent. proof stress (tons per sq.in.)	Elongation* (per cent.)	U.T.S. (tons per sq.in.)	0·1 per cent. proof stress (tons per sq.in.)	Elongation* (per cent.)
1	18·1	10·0	14	34·0	29·7	10·5
2	†	†	†	35·5	31·0	8·5
3	15·9	9·1	13	35·6	31·2	9·9
4	†	†	†	38·0	29·5	13·5
5	18·4	9·3	14	35·6	30·8	9·9

* Percentage elongation on 2 in.
† Insufficient material for testing.

tible to cracking at temperatures below the solidus, and in order to differentiate this from cracking at temperatures above the solidus in these alloys some welds in each alloy were allowed to cool down to room temperature in the restraining jig, while others were removed from the jig after the occurrence of cracking at temperatures above the solidus and before any sub-solidus cracking. In all cases, the total length of cracking on both sides of the welded sheets after removal from the jig was measured. The amount of cracking at temperatures both above and below the solidus in any one alloy was obtained by measuring the average amount of cracking in sheets of that alloy cooled to room temperature in the restraining jig. The amount of cracking at temperatures above the solidus only was determined from measurements of the total length of cracking in sheets removed from the jig before sub-solidus cracking occurred. The difference in the two results for any one alloy gave the length of cracking at temperatures below the solidus.

With the exception of alloy 2, welded in the annealed condition, the five alloys containing copper were considerably more susceptible to cracking than the alloys free from copper—as will be seen by a comparison of Tables VIII and IX with Table IV. The total lengths of cracking obtained in the restrained weld tests of the five alloys were similar (Tables VIII and IX, column 3), but the alloys were very different in their susceptibility to cracking at temperatures below the solidus. With the alloys welded in the annealed condition there was apparently some relationship between the copper content of the alloys and their tendency to crack at temperatures both above and below the solidus. There was a general tendency for the susceptibility to cracking at temperatures above the solidus to be high with a copper content of up to about 1 per cent. and then to decrease rapidly with further increase in copper. This result would be expected from general theoretical considerations based on results obtained previously with other alloy systems. Alloy 2 was somewhat anomalous in this respect, but it is worthy of note that in this alloy the magnesium, iron and silicon contents were appreciably higher than in the other alloys. As would be expected from previous results,[2] the susceptibility of the annealed alloys to cracking at temperatures below the solidus—which, in alloys susceptible to sub-solidus cracking is dependent to some extent upon the amount of cracking which occurs at temperatures above the solidus—increased with increase in the copper content.

There was no obvious relationship between the copper contents of the alloys welded in the fully heat-treated condition and their tendency to crack when welded. The reason for the difference in behaviour of the same alloys when welded in the annealed and in the heat-treated condition

Table VII.—Mechanical Properties of Welds in Alloys Listed in Table V

Alloy Ref. No.	Results of Tests on Welds in Annealed Material				Results of Tests on Welds in Heat-treated Material			
	Total Length of Cracking in Restrained Weld Test* (in.)	U.T.S. (tons per sq.in.)	Elongation† (per cent.)	Position of Fracture	Total Length of Cracking in Restrained Weld Test** (in.)	U.T.S. (tons per sq. in.)	Elongation† (per cent.)	Position of Fracture
1	7·6	18·9	5	In basis metal‡ or at edge of weld	7·0	18·4	4	In weld or at edge of weld
2	3·4	14·8	1	In weld or in basis metal	6·8	16·4	1	In weld
3	8·6	17·4	3	In weld, at edge of weld, or in basis metal	8·2	15·2	2	In weld or at edge of weld
4	7·3	17·7	2	In weld	7·7	15·9	2	In weld
5	5·6	13·3	1	In weld or at edge of weld	8·5	10·3	1	In weld or at edge of weld

* See Table VIII. ** See Table IX. † Percentage elongation on 2 in.
‡ In basis metal indicates that the fracture occurred at a point remote from the heat affected zone of the weld.

is not clear.

Cracking in the welds occurred somewhat indiscriminately at the centres and at the edges of the welds, and occasionally at both positions simultaneously. The location of the cracking in the welds appeared to be independent of the composition of the material and of its condition before welding. In certain of the alloys the cracks which occurred at temperatures above the solidus during restrained weld testing were located in the parent metal some short distance from the weld bead. The occurrence of cracking above the solidus at such points must be ascribed to the long solidification range of the alloys.

No great difficulty was experienced with any of the five alloys in depositing the weld bead; there was, however, some slight tendency for the weld metal to sink through.

The majority of the welded sheets showed no tendency to blistering in the vicinity of the weld bead, although there was some slight tendency for surface blistering to occur on the under-surface of three or four of the welded sheets. Microscopic examination of the welded plates of the six alloys indicated that there was very little porosity in the alloys. As mentioned earlier, the alloys of varying zinc and magnesium contents and free from copper were observed to be particularly susceptible to surface blistering and internal porosity when welded; the freedom from surface blistering and porosity of the five alloys containing copper may possibly be ascribed to their relatively low magnesium contents and to the good surface quality of the sheets used for the welding tests. The high surface quality of the sheets was, no doubt, a result of the material being manufactured under large scale industrial conditions.

Table VIII.—Results of Restrained Weld Tests on Alloys listed in Table V in the Annealed Condition

Alloy	Average Length of Cracking at Temperatures above the Solidus in Restrained Weld Test (in.)	Average Total Length of Cracking at Temperatures above and below the Solidus in Restrained Weld Test (in.)	Average Length of Cracking at Temperatures below the Solidus in Restrained Weld Test (in.)*
1	7·6	7·6†	0
2	2·1	3·4	1·3
3	5·9	8·6	2·7
4	1·7	7·3	5·6
5	1·0	5·6	4·6

* Difference of the figures in Columns 2 and 3.

† This alloy did not crack at sub-solidus temperatures when allowed to cool in the restraining jig after welding and hence no weld tests were carried out in which the test plates were removed from the restraining jig immediately after solidification of the weld bead.

Table IX.—Results of Restrained Weld Tests on Alloys listed in Table V in the Fully Heat-Treated Condition

Alloy	Average Length of Cracking at Temperatures above the Solidus in Restrained Weld Test (in.)	Average Total Length of Cracking at Temperatures above and below the Solidus in Restrained Weld Test (in.)	Average Length of Cracking at Temperatures below the Solidus in Restrained Weld Test (in.)*
1	7·0	7·0†	0
2	0	6·8	6·8
3	3·8	8·2	4·4
4	1·4	7·7	6·3
5	6·3	8·5	2·2

* See footnote to Table VIII.

† See footnote to Table VIII.

The greatest degree of blistering during welding occurred with alloy 1 and a section through a weld made between two heat-treated sheets of this alloy is shown in Fig. 4. In this weld the surface blistering is associated with slight sub-surface porosity and a small amount of porosity at the side of the weld bead. Sub-surface porosity dispersed as in Fig. 4 would, however, be removed in any surface machining operation subsequent to welding and no pronounced effect on the mechanical properties of the weld would be expected.

DISCUSSION

From the results recorded in Table IV it will be seen that the copper-free aluminium-zinc-magnesium alloys within the range of compositions considered, and particularly those alloys containing not more than 4 per cent. magnesium, had good welding properties and no pronounced tendency to crack during welding. This result is somewhat at variance with the results previously obtained with copper-containing alloys to specifications DTD 363A and DTD 687. A comparison of the results obtained in restrained weld tests of the alloys free from copper (Table IV) and those obtained in tests of the alloys containing copper (Tables VIII and IX) suggests that the presence of copper may have a detrimental effect on the resistance of aluminium-zinc-magnesium to crack during welding. In support of this suggestion, those workers[4, 17] who have reported poor welding qualities for aluminium alloys containing zinc and magnesium have examined only alloys containing copper, the alloy examined by Vachet containing 1·5 per cent. copper and that examined by Pendleton 0·5 per cent. copper. The results obtained by these workers agree with the results obtained with the aluminium-zinc-magnesium alloys containing 1·32 and 0·44 per cent. copper recorded in Tables VIII and IX. An indication of the probable effect of copper on the welding properties of aluminium-zinc-magnesium alloys may be obtained from the work of Saulnier[18] who investigated the hardening process in quenched aluminium-zinc-magnesium alloys both free from and containing copper. Saulnier found that with the alloys containing copper the hardening could be attributed to the presence of $CuAl_2$ which has the effect of increasing the elastic limit and ultimate tensile strength, and decreasing the ductility, of aluminium alloys containing zinc and magnesium. Such decrease in the ductility of an alloy may

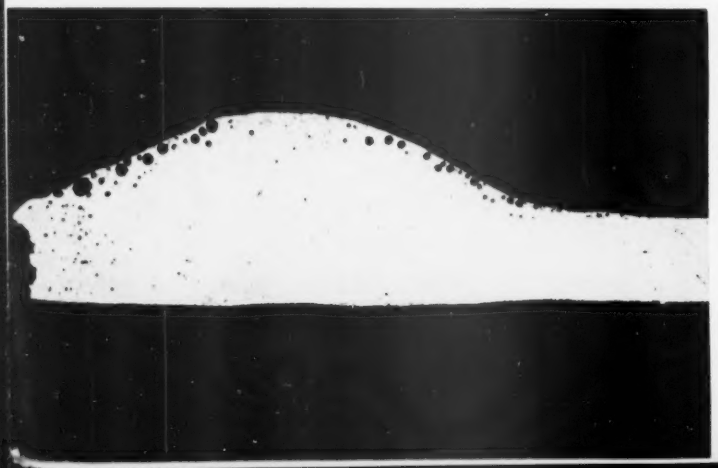


Fig. 4. Section through weld made in alloy 1, in fully heat-treated condition. Unetched. ×6.

increase the tendency for that alloy to crack, during welding, at temperatures in the region of the solidus, or at considerably lower temperatures.

The presence of elements other than copper may also have an effect on the tendency to cracking. Thus Patterson[19] reported that the addition of 0·3 per cent. chromium or 0·1 per cent. vanadium, to aluminium-zinc-magnesium alloys containing 0·5 per cent. manganese, caused a deterioration in their working properties; the alloys containing such additions being subject to cracking when continuously cast and the billets tending to burst when hot-rolled. It is of interest that the first of the two basic alloys used in the preliminary ring casting tests of aluminium-zinc-magnesium alloys of commercial purity (Table I) contained 0·46 per cent. manganese, while the second contained 0·1 per cent. manganese and 0·25 per cent. chromium. The alloy examined by Vachet[4] contained 0·25 per cent. chromium. Mott and Thompson,[7] however, suggest that chromium has no marked effect on the mechanical properties of aluminium alloys containing zinc and magnesium, but that additions of manganese decrease the ductility of such alloys while increasing their hardness, proof stress and ultimate tensile strength. A comparison of Tables IV, VIII and IX suggests that copper is the prime agent in determining the susceptibility of aluminium-zinc-magnesium alloys to crack during welding, and it would seem desirable to maintain the copper content as small as possible in aluminium-zinc-magnesium alloys for which the best welding properties are required. The exclusion of copper from such alloys apparently has no markedly detrimental effect on their mechanical properties.

The results obtained with the aluminium-zinc-magnesium alloys free from copper suggested that welds in alloys of this type would be particularly susceptible to superficial blistering and internal porosity; but welds made in the five commercial alloys containing 0·44 to 2·04 per cent. copper contained very little porosity or surface blistering. The freedom from such defects of the five alloys containing copper may possibly be ascribed to their relatively low magnesium contents (approximately 2·5 per cent.) or to the good surface quality of the sheets used for the welding tests compared with that of the sheets used in the tests of the alloys free from copper. This latter factor is probably associated with the difference in mode of manufacture of the alloys free from copper and those containing copper, the alloys free from copper having been made under laboratory conditions and those containing copper under large scale industrial conditions. To minimise the possible occurrence of blistering and porosity in the weld area with aluminium-zinc-magnesium alloys, it would seem generally desirable to maintain the magnesium content of all alloys of this type which are to be used for welding at not more than 4 per cent. It is, however, inadvisable to reduce the magnesium content appreciably below about 4 per cent. because the tendency for weld cracking increases as the magnesium content decreases at any one zinc content.

Experience has shown that weld cracking is unlikely in industrial welding conditions with alloys which give up to 1 in. of cracking in the restrained weld test. For applications where alloys having high strength and reasonably good welding properties are required, the aluminium-zinc-magnesium alloys free from copper would seem very suitable. For the best welding properties, as regards freedom from cracking, blistering and porosity, together with high strength and reasonable ductility after welding, alloys in this system containing 4 per cent. zinc and 4 to 5 per cent. magnesium welded in either the annealed or heat-treated condition, or an alloy containing 6 per cent. zinc and 4 per cent. magnesium welded in the annealed condition, should be most satisfactory. These alloys also have acceptable casting and rolling properties.

CONCLUSIONS

The aluminium-zinc-magnesium alloys free from copper which have been examined are very susceptible to heat treatment and would seem to be suitable for applications where high strength and reasonably good welding properties are required. The copper content of aluminium-zinc-magnesium alloys appears to be of prime importance in determining the susceptibility of such alloys to cracking during welding, and it would seem desirable to maintain the amount of this element as low as possible in aluminium-zinc-magnesium alloys in which the best welding properties are required. The exclusion of copper from such alloys appears to have no markedly detrimental effect on their mechanical properties although further work may, perhaps, indicate that the presence of copper is desirable from the aspect of commercial production.

To minimise the occurrence of blistering and porosity in the weld area, it is suggested that the magnesium content of aluminium-zinc-magnesium alloys to be used for welding should be maintained at not more than about 4 per cent. Reduction of the magnesium content much below about 4 per cent. is not advisable because the tendency to weld cracking in aluminium-zinc-magnesium alloys increases with decrease in the magnesium content at any one zinc content.

For applications where aluminium alloys are required, having satisfactory welding properties and high strength both in the heat-treated condition and after welding aluminium-zinc-magnesium alloys free from copper and containing 4 per cent. zinc and 4 to 5 per cent. magnesium, should be suitable welded in either the heat-treated or annealed condition. An alloy containing 6 per cent. zinc and 4 per cent. magnesium welded in the annealed condition would also appear to be satisfactory. These alloys have casting and rolling properties at least as acceptable as those of commercial alloys in this series.

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THE MECHANICS OF NOTCH BRITTLE FRACTURE

A short introduction was given in Welding Research, August 1952, *to a new approach which had been made to the subject of Notch Brittle Fracture by Dr. A. A. Wells. It is now possible to publish a full account of the work which has been carried out, up to December*, 1952.

By A. A. WELLS, Ph.D.

SUMMARY

Notch brittle fracture at temperatures just below the transition temperature, in material showing usual ductility in the tensile test, with statically applied external loads, is shown to occur when these external loads are sufficient to satisfy two separate conditions of initiation and propagation.

In structures possessing no residual systems either of plastic strain or high elastic stress, the initiation loading condition is shown to be that of general yield, where a plastic zone entirely crosses the body, and embraces the root of a notch. Loads sufficient to cause this are generally higher than those met in service. When residual stress systems either of plastic strain or high elastic stress are present due to the local heating and cooling from fusion welding, brittle fractures may be started from notches or hot cracks in commercial mild steel, below the transition temperature, with small external loads or even none at all.

A method is described which makes it possible to measure the energy absorbed in plastic flow along the surfaces of a propagating brittle fracture, by means of the temperature wave created. A notch brittle material has been shown experimentally to possess, at a given temperature, a minimum surface energy of crack propagation, as first suggested by Irwin. Thus, if a fracture is initiated in a structure, but the rate of release of stored elastic energy with increase of length of the crack is insufficient to provide the minimum surface energy of the material, then the crack will not propagate. On the other hand, the surface energy of the material can have values above the minimum, so that the type of brittle breaking strength formula based on surface energy, and first proposed by Griffith, cannot be used indiscriminately. In general, it appears that the brittle breaking loads of large specimens are determined by the initiation process, where the influence of stresses other than those due to external loads is excluded, while the breaking loads of small specimens are controlled by minimum surface energy.

These results have been applied to explain certain notch brittle fractures which have recently occurred in large welded structures. In some cases these fractures were initiated in butt welds, and propagated transversely in the material on either side. At the present time this type forms a fair proportion of the total number of fractures recorded. The initiation of the fracture has been shown to occur because of the coincidence of an initial crack and plastic stretching, in the direction of a weld, of the adjacent material due to local heating and cooling; the first stages of crack propagation rely on a residual stress field of yield point magnitude which is the end result of this stretching. The later stages of catastrophic propagation then depend on the lower service stress in the whole structure. Although residual stress relief, or the removal of macroscopic weld cracks, would seem to be adequate to prevent this form of failure under the limited conditions discussed above, where the structure is statically loaded, it is obvious that other metallurgical and practical factors must be considered before any positive recommendations can be made to cover all cases.

INTRODUCTION

The ductility and accommodating manner of commercial mild steel is so well known that it is surprising to find that failures of a brittle nature do occur in even fairly thin material. Fortunately, these failures are rare, but they may be catastrophic. Four European bridges have collapsed due to brittle fracture, two or more United States tankers have broken their backs in still water, and recently several oil-storage tanks have burst in Britain and elsewhere. In all these cases, the external loads on the structures at the time of failure were not only entirely static, but were also considerably less than the structures themselves were designed to withstand. Other features which these cases had in common were, firstly, that the components of the structures were joined by fusion welding; secondly, that the atmospheric temperatures at the time of failure were low; thirdly, that some variety of notch effect could always be traced as the source of fracture; and fourthly, that there was little evidence of ductility in the fracture zone. In addition to these clearly defined examples of complete failure due to brittle fracture, there have been more where brittle failure was partial and where the causes could be traced, for instance, to gross structural weakness, to excessive or impact loading, etc. It is for the first mentioned cases of low static loading that an adequate explanation of failure is most urgently required.

According to Orowan,[1] notched bar bending tests were described by Tetmajer (Switzerland) in 1884, and by Tunner (Austria) in 1885,* so that notch brittleness is not a recently observed phenomenon. In these early tests the specimen was simply broken by a hammer blow, and the brittleness recognised by lack of sufficient plastic deformation, and by the crystalline appearance of the surface of fracture.

Controlled impact testing was introduced at about the end of the nineteenth century and soon after this the dependence of the type of fracture on the temperature of testing was realised. Batson and Hyde[2] certainly recognised in 1922 that the change from ductile to brittle fracture took place between 17 deg. C. and −40 deg. C. for some mild steels. Ludwik[3] (1926) and his school soon provided an explanation for this remarkable behaviour. Salmon[4] (1931) summarised their views on the effect of notches and discontinuities as follows: "Material in which the resistance to shear (transcrystalline slip) is relatively large, fails when the stress

* Attention has also been drawn to the publication by Kirkaldy,[49] preceding the above by twenty-two years.

exceeds the cohesive force between the grains; there is little or no deformation and the material is said to be brittle. In ductile material the resistance to shear is relatively small, and deformation due to slip occurs when the shear exceeds that resistance. The resistance to slip increases with the rate of deformation; further, the relative values of the cohesive and shear resistances can be modified by heat treatment, and depend on the temperature of the test.

In a notched bar test a small volume of highly stressed material near the notch has to absorb the energy. This volume is surrounded by less highly stressed material which prevents plastic flow, that is, it prevents the lateral distortion which accompanies the longitudinal strain, and in consequence a state of three-dimensional stress is set up. In one-dimensional stress the maximum shear stress is one-half the direct stress, but in three-dimensional stress the maximum shear stress is one-half the difference between the greatest and least principal stresses. These stresses may therefore have large values before the resistance to shear is reached, and ere that, the cohesion may be overcome. Not only so, but since the highly stressed volume which absorbs the energy is small, the rate of deformation will be large, even at relatively low impact velocities; this will raise the resistance to shear, enhancing the tendency to cohesion failure. Apparently brittle fractures in ductile materials are thus explained.

The type of fracture depends, therefore, on (1) the amount of restriction to plastic flow, (2) the rate of deformation, and (3) the temperature of the test."

Ludwik's explanation is acceptable, even to-day, although we now know that his cohesion is not intergranular, but, in fact, transgranular on another crystal plane from the shear plane.[5] Failure on this cleavage plane appears to depend, as Ludwik suggests, on a limiting tensile stress. Cleavage strength is not easily measured, but if the relative strengths are considered of a plain tensile specimen at the temperature of liquid air, and a deeply notched bar just below its transition temperature, near 0 deg. C., both of which fail in the cleavage mode, it is evident that there is no great variation of this strength with temperature. The shear strength, on the other hand, distinctly increases with drop of temperature. The change from ductile to brittle failure in individual crystals, as the temperature drops, can thus be adequately explained. Objections are sometimes raised as to the definition of the term "shear strength." However, when it is realised that yield point, ultimate strength and true fracture stress, in the common tensile specimen, all tend to increase with decrease of temperature until brittleness sets in, the general validity of the Ludwik explanation may be seen to be unimpaired.

Mention should also be made of the behaviour of certain materials, such as glass, which show practically no plasticity at normal temperatures, and invariably fail by cracking. These materials are elastic in that their deformation under load is reversible, and proportional to the applied load. After a certain load is reached, fracture takes place by means of a crack emanating at high speed from some flaw or discontinuity. As in the brittle fracture of mild steel, for which speeds up to 6,000 ft. per second have been measured,[6, 7] the speed of the crack is often so great that there is no opportunity for extra work to be done by the external forces while propagation takes place. In this case, energy needed to create the fresh surface area at the crack must come from that stored elastically in the remainder of the body which is available on relaxation of the stress. Solid materials possess surface tension, just as do liquids. Griffith[8] was first to show that an energy balance can be struck between these two sources of supply and demand at the instant of fracture. Griffith's estimate of the available elastic energy is based on a calculation by Inglis[9] of the complete elastic stress distribution in a stretched plate containing an elliptical hole. The extreme case of the elliptical hole occurs when the minor axis vanishes and the hole becomes an internal crack. From the stress distribution in the plate, the total elastic potential energy may readily be determined by integration. The next step is to determine the change in potential energy with unit increase of length of the crack. This is then equated to twice the surface tension, since unit length increase of the crack gives rise to twice unit area of fresh surface, for unit plate thickness. The result is a simple expression for the critical stress σ, applied to the ends of the plate, which will just cause the crack to extend. The expression is

$$\sigma = \sqrt{\frac{4ET}{\pi l}} \quad \ldots \ldots \ldots \ldots \quad (1)$$

where E is Young's modulus, T is the surface tension and l the initial crack length. Griffith was also able to predict that stretching the plate in a direction parallel to the crack would not cause the crack to extend at any applied stress. He conducted experiments with glass which gave triumphant proof of the correctness of the calculation. To do this, a measure of the surface tension was necessary. Fortunately, this was known not to vary greatly with temperature, for physical reasons, and an extrapolation from measurements in the liquid phase was used.

If Griffith's expression is used to predict failing loads of internally notched steel plates in the notch brittle condition, surface tensions 10,000 times greater than the independently measured values must be assumed, in order to account for observed breaking loads. Irwin[10] has recently suggested that the slight amounts of plastic flow occurring all along brittle fracture zones might account for this apparent extra energy absorption. If this were so, a steel might be expected to possess an intrinsic surface energy for brittle fracture, which could be used for the brittle condition, just as yield point and ultimate strength are used to predict behaviour in the ductile condition.

THE INITIATION OF NOTCH BRITTLE FRACTURES

The fracture condition proposed by Griffith and Irwin may be checked by testing a series of geometrically similar internally notched flat plates in static tension, provided that the plates all come from the same melt and are rolled to the same thickness, in order to minimise metallurgical variations. Boodberg and collaborators[7] and Wilson and collaborators[11] have published the results of such tests, with $\frac{3}{4}$ in. plates from 12 in. to 72 in. wide, with a central saw cut perpendicular to the direction of tension and one-quarter the width of the plate. In all cases the ends of the saw cuts were finished with a jeweller's saw, cutting to 0·01 in. width. Both teams of investigators tested plates of each width, using several steels whose analysis and mechanical properties are summarised in Table I, and at different temperatures, to establish the effect on breaking load of the transition from ductile to brittle behaviour. For each width, regardless of variations of steel composition and temperature, the general trend of breaking loads was similar. Brittle breaking loads were somewhat lower than the ductile values, and individual steels showed correlation between tensile strengths of notched and plain specimens. Both sets of results obtained at temperatures just low enough to show complete brittleness, and for all steels represented in both 12 in. and 72 in. widths, may be averaged for each width if comparison is required. For nine steels, the ratio of the *net** average breaking stress at 12 in. width, to that at 72 in. width, is 1·16, whereas if equation 1 were applicable, and the breaking strength were controlled by surface energy, the ratio should be $\sqrt{\frac{72}{12}} = 2{\cdot}45$.

If it is argued that true geometrical similarity is not

* *Net* average breaking stress is defined as breaking load divided by the minimum area at the notched cross-section.

Table I.—Properties and compositions of steels relating to this investigation.

Investigator	Thickness (in.)	Material Designation	Mechanical Properties				Chemical Constituents (per cent.)							
			Yield Point (tons per sq.in.)	Ultimate (tons per sq.in.)	Reduction of Area (per cent)	Transition Temperature (deg. C.)	C	Mn	Si	P	S	Ni	Cr	Others
Tipper	0·339	Fractured plate U.S. Samzona	20·6	28·0	45·0	15	0·20	0·38	<0·02	0·038	0·034	Trace	0·10	Cu = 0·10
	0·410	S.D. 413	17·0	26·5	61·0	−20	0·18	0·33	0·05	0·022	0·040	,,	Trace	Cu = 0·20
	0·560	1. Basic O.H.	16·2	28·4	59·0	−20	0·19	0·58	0·07	0·010	0·028	0·13	0·05	Cu = 0·08
	0.560	5. Acid O.H.	16·8	28·0	48·0	0	0·17	0·41	0·06	0·050	0·039	0·14	0·05	Cu = 0.10
	0·510	4. Acid O.H.	19·4	32·0	49·0	−10	0·22	0·72	0·17	0·039	0·045	0·21	0·09	Cu = 0·13
Bagsar	¾	12.	16·5	27·0	—	>22	0·33	0·55	0·02	0·014	0·035	—	—	—
	1¼	13.	13·9	25·8	—	>22	0·26	0·46	0·01	0·015	0·030	—	—	—
	1	15. Semi-killed	13·6	26·5	—	46	0·20	0·45	0·07	0·019	0·029	—	—	—
	1	17. Semi-killed	14·7	28·8	—	27	0·19	0·42	0·07	0·015	0·030	—	—	—
	1	18. Semi-killed	14·0	27·4	—	40	0·20	0·63	0·07	0·020	0·033	—	—	—
	¾	21. Rimmed	19·7	29·4	—	−23	0·21	0·43	0·02	0·010	0·028	—	—	—
	15/16	23. Semi-killed	14·9	28·3	—	43	0·25	0·45	0·05	0·015	0·024	—	—	—
Thomas and Windenburg	¾	A Semi-killed	16·1	26·8	57·5	0	0·26	0·50	0·03	0·012	0·039	0·02	0·03	—
	¾	B_r Semi-killed	14·9	25·5	64·5	−11	0·18	0·75	0·07	0·008	0·030	0·05	0·03	—
Carpenter and Roop	¾	B_n Semi-killed normalised	15·6	25·5	63·5	−2	0·18	0·73	0·04	0·011	0·030	0·06	0·03	—
	¾	C Semi-killed	16·1	29·0	56·5	34	0·24	0·48	0·05	0·012	0·026	0·02	0·03	—
	¾	D_r Killed	16·8	29·0	53·9	−10	0·22	0·55	0·21	0·013	0·024	0·16	0·12	Cu = 0·22
Boodberg and collaborators	¾	D_n Killed normalised	15·5	26·6	59·0	−10	0·19	0·54	0·19	0·011	0·024	0·15	0·12	Cu = 0·22
	¾	E Rimmed	13·4	25·4	56·4	27	0·20	0·33	0·01	0·013	0·020	0·15	0·09	Cu = 0·18
Wilson and collaborators	¾	H Killed	15·9	28·3	63·5	−15	0·18	0·76	0·16	0·012	0·019	0·05	0·04	Al = 0·053
		N Alloy	25·8	35·7	64·5	−50	0·17	0·53	0·25	0·011	0·020	3·39	0·06	Al = 0·077 Cu = 0·19
Robertson	⅜ and 1	A, C and D	—	—	—	7 to 20	0·20/0·31	1·02/1·44	—	—	—	—	—	—
Wells	3/16	Spring	36·0	55·5	26·0	>20	0·54	0·87	2·01	0·025	0·035	0·06	0·03	—
	⅜	Ship	19·8	28·6	59·2	−15	0·20/0·30*	—	—	—	—	—	—	—
	1¼	Structural	17·9	27·6	54·0	>20	0·185	0·57	0·05	0·023	0·030	—	—	—

* Estimated from micrograph.

achieved by using a saw cut notch 0·01 in. wide in each case, then it follows that the 72 in. plates have relatively too sharp a notch, when compared with those 12 in. wide. To correct this would reduce the actual strength ratio of 1·16 and make the theoretical comparison even more in error.

Although the Griffith-Irwin mechanism may still control brittle crack propagation, these experiments show that it does not control the stress necessary for initiation. However, the experiments described above suggest that the net average brittle strength is closely related to the yield point. For the 72 in. plates this net average brittle strength for all the steels is 17·8 tons per sq.in. The average tensile yield point to be compared with this, is 16·8 tons per sq.in. The comparison merits further consideration if a criterion for the initiation of notch brittle fracture is to be obtained.

Plastic Theory

Frequent attempts have been made to explain notch brittleness in terms of the stresses at the root of a notch in completely elastic material. The theory for the stress distribution at such a notch in a stretched plate has been given by Inglis[9] for an internal notch, and by Neuber[12] for both shallow and deep external notches. In general, the maximum elastic tensile stress at the root of a sharp notch is a multiple of $\sqrt{l/\rho}$ times the tensile stress remote from the notch. ρ is the radius of curvature at the root, and l is the length of the notch. It follows from this that impossibly high stresses would exist at the roots of sharp notches if the material remained elastic. In fact, it is obvious from the experiments described above[7,11] that extensive fully plastic zones must always be present, and especially at the roots of notches, to account for such high breaking strengths, even in the fully notch brittle condition. It is therefore necessary to review present knowledge of plasticity in order to progress.

For plasticity calculations it is usual to assume that the material is elastic up to a definite yield point, after which, for further increase of strain, the stress remains constant. Strain hardening is thus ignored, and in any case, for the mild steels being considered,which show a definite yield point, it is usually absent until strains exceed the order of 1 per cent.

There are broadly two methods of attack in plasticity problems. In the first, elastic strains are neglected and the material is assumed to be rigid and plastic in the respective zones. This method is simple to apply, but will only deal accurately with what is known as fully-developed plasticity. In addition, with this method, under multi-axial stress, the maximum shear criterion is usually accepted. Under these conditions, yield at a point first occurs when the difference there between the greatest and least principal stresses reaches the tensile yield point. This introduces a simplification into plate problems, because if deformation normal to the plate is prevented (plane strain), it can be shown that the greatest and least principal stresses in the plastic zones lie in the plane of the plate. In plates so thin that no stresses can exist through the thickness (plane stress), the situation in the plastic zones is less simple, because shearing may take place between successive laminar planes through the thickness, as well as along those normal to the plane of the plate.

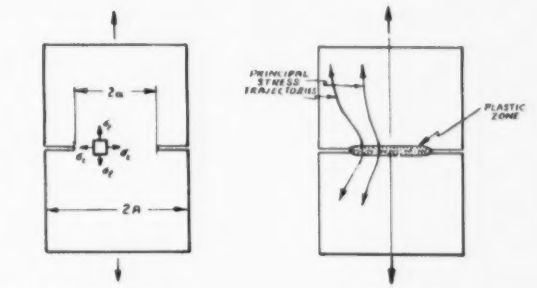
The other approach is more powerful, but also more complicated, and makes use of a relaxation method of calculation, first applied to the problem by Allen and Southwell.[13] The material is assumed to be elastic/plastic, so that all stages of the growth of plastic zones may be observed. Here, the maximum shear strain energy criterion of yield is employed.

Both methods have been applied to the solution of the externally notched bar problem, with axial tension or compression. In this case the bar is initially of uniform thickness, and constant external width 2A. Two symmetrical narrow slits are made in the edges of the plate, normal to the direction of pull, so as to leave an un-notched width 2a (Fig. 1a). On the narrowest section through the notch, the longitudinal and transverse stresses will be defined as σ_l and σ_t respectively. Due to the symmetry of the section they will also be principal stresses. It is frequently the object of notched bar plasticity studies to determine the restraint factor. This is defined as the net average value of σ_l across the notched zone when the plastic zones are fully developed, divided by the yield point of the material.

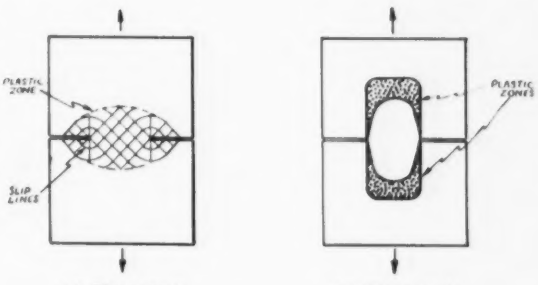
A formal solution for the notched bar plastic stress distribution corresponding to plane stress is not available in the literature. In spite of this, two simple deductions are possible. Firstly, σ_l and σ_t will be of the same sign on the notched cross section, since the principal stress trajectories may be estimated to curve outwards from the axis of the plate, when passing away from this section (Fig. 1b). From this, the second deduction follows that, since the principal stress normal to the plate is always zero, the value of σ_l cannot exceed the yield point. When general yield has occurred, and the plastic zone is as shown (Fig. 1b), the average stress will also be equal to the yield point, and the restraint factor will be 1.

When the thickness of the bar becomes great compared with the notched width, a plane strain solution is more applicable. Prandtl[14] (1920) postulated the very simple slip line field in the plastic zone which appears to satisfy the case where the ratio A/a is large (Fig. 1c). Three years later Hencky[28] calculated the plastic stresses corresponding with this field. Hill[15] has recently corrected Hencky's calculation of the stresses in the plastic zone, and finds for fully established plastic zones that the values of σ_l and σ_t are $(1+\frac{\pi}{2})$ and $\frac{\pi}{2}$ times the yield point and are respectively, uniform across the notched zone.

The remaining plane strain solutions for notched bars are due to Jacobs,[16] and have been computed for A/a = 2,



(*a*) General. (*b*) Plane stress.



(*c*) Plane strain, A/*a* large. Prandtl-Hencky-Hill solution.

(*d*) Plane strain. A/*a* = 2, 4 and 8. Jacobs relaxation solution.

Fig. 1. Plastic flow in externally notched plates.

4 and 8, by means of the relaxation method. The results are given in Table II for the incidence of full plastic zones. The relaxation solutions show σ_1 to be concentrated slightly at the notch roots, as compared with the centre of the bar.

Table II.—Restraint factors for externally notched plates.

	A/a	σ_1 at root (÷ Yield Point)	Average Net Stress across Notched Zone (÷ Yield Point) = Restraint Factor
Jacobs	2	1·75	1·38
	4	2·53	1·90
	8	3·44	2·61
Prandtl-Hencky-Hill	∞	2·57	2·57

The comparison shown in this table between the Jacobs and Prandtl-Hencky-Hill restraint factor solutions is very good. Jacobs shows the plastic zone to develop as in Fig. 1d.

No plasticity calculations appear to be available for internally notched bars, but it is feasible that, just as in the elastic cases, the general trends of results would be similar to those for externally notched bars.

Comparison with Notched Bar Tests

A closer comparison may be made between the breaking strengths of axially loaded notched bars and the yield points now that the conditions for general yield are established. In order to do this, the results of six investigators,[7, 11, 17, 18, 19, 20] have been abstracted for the steels whose compositions and mechanical properties are summarised in Table I. These notched bar results are given in Table III. All the results represent testing temperatures sufficiently low to give 100 per cent. brittle fractures; 177 individual results were analysed at the 64 conditions given in Table III. To show the significance of these results, Fig. 2 has been plotted for averaged results at each of the 64 conditions. Average net breaking stress, in terms of the tensile yield point,* is plotted against the ratio of thickness to net width. This ratio distinguishes between plane stress at a ratio of zero, and plane strain at a ratio of infinity, although a ratio of unity would probably represent almost complete plane strain. All the results, with one slight exception, lie above the yield point. Moreover, since the largest ratio A/a in these specimens is 1·33, apart from the deliberately widely notched plates of Thomas and Windenburg,[19] the expected restraint factors in plane strain would be less than 1·13. Those results in Fig. 2 for which the ratio thickness to width is largest, and for which plane strain might be the expected condition, seem to show lower limits of the net breaking stress/yield point ratio tending to 1·13.

The results of Thomas and Windenburg for A/a = 4 show net average breaking strengths of about 1·6 times the yield point, whereas the plane strain restraint factor would be 1·9, if calculated for external notches. A restraint factor calculation for an equivalent internal notch would almost certainly give a lower value, because of the proximity of the ends of the notches to the outer sides of the plate, so limiting the transverse stress across this zone. In any case, these plates would be likely to approach the condition of plane stress, and the investigators quote elongations to fracture of 0·03 in.-0·15 in. measured over 23 in., so that general yield must have occurred before fracture.

The preceding results have all been associated with axial loading. It is therefore interesting to observe the behaviour of the so-called cleavage-tear test specimens, in which the load is deliberately applied to the specimen through pin joints at a definite and large eccentricity. Bagsar[21] has published results of tests with different steels at temperatures giving brittle fractures, and with ratios of the eccentricity of loading e to the specimen width d, varying from 0 to 2·2 (Fig. 3). The results are published as breaking stresses on the net area, given by the static breaking load P divided by the net area of cross-section. Now, an eccentric load of this type will almost certainly create a plastic hinge through the

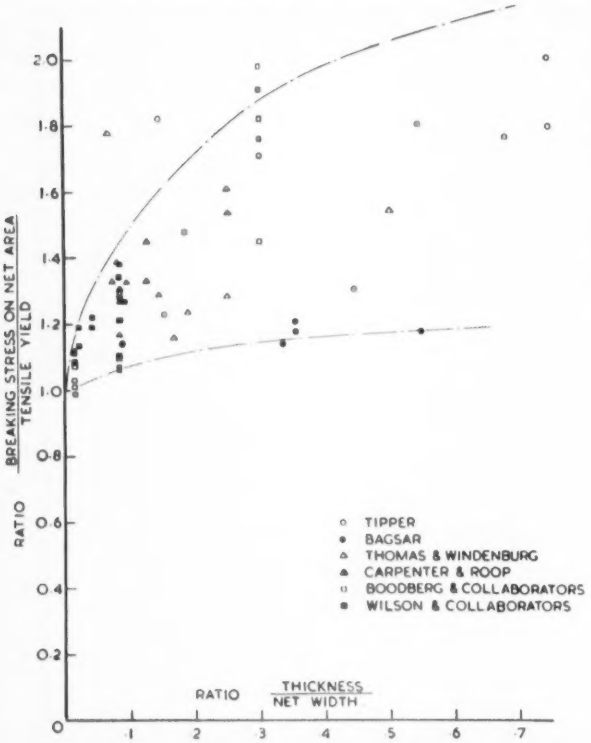


Fig. 2. Static tensile brittle breaking strength of axially loaded notched mild steel plates.

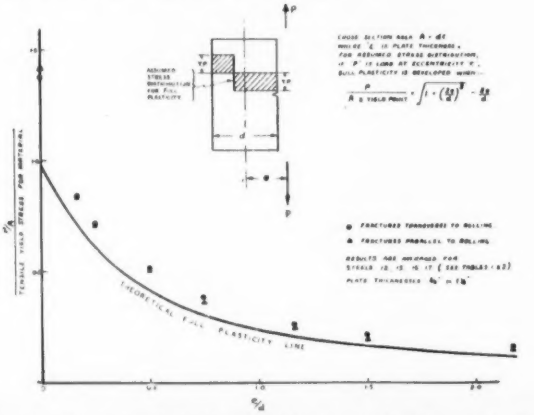


Fig. 3. Static tensile brittle breaking strength of eccentrically loaded externally notched mild steel plates.

* Except where otherwise stated, room temperature values have been used, since, in general, the transition temperatures quoted are equally above and below room temperature. It is unlikely that the transition temperature yield point values will have been more than ± 5 per cent. different from those measured at room temperature, even in the extreme cases.

Table III.—Collected results of tensile tests of artificially notched mild steel plates in the notch brittle condition.

	Investigators	Material (see Table I)	Thickness (in.)	Gross Width (in.)	Notch Depth (in.)	Net Width (in.)	Notch Type	Notch Root Radius (in.)	Number of Specimens	Net Average Breaking Stress (tons per sq.in.)	Ratio: Thickness / Net Width	Ratio: Breaking Stress / Yield Point
1	TIPPER	Samzona	0·339	1		0·75			4	27·0	0·446	1·31
2		,,	0·339	3		2·81			5	25·3	0·153	1·23
3		SD 413	0·410	1		0·75			6	30·8	0·547	1·81
4		SD 413	0·410	3	¼	2·81	Ext.	0·010	2	30·9	0·145	1·82
5		1	0·560	1		0·75			2	32·5	0·747	2·01
6		5	0·560	1		0·75			3	30·2	0·747	1·80
7		4	0·510	1		0·75			3	34·3	0·680	1·77
8		4	0·510	3		0·75			4	28·7	0·185	1·48
9	BAGSAR	15	1	3		2·81			5	16·1	0·356	1·18
10		17	1	2		1·81			6	17·3	0·551	1·18
11		18	1	3	3/16	2·81	Ext.	0·0015	5	17·0	0·356	1·21
12		21	¼	3		2·81			3	22·4	0·089	1·14
13		23	15/16	3		2·81			7	17·0	0·337	1·14
14	THOMAS & WINDENBURG	E		12	0·375	11·62			4	23·8	0·067	1·78
15		E		12	2·64	9·36			4	18·6	0·080	1·39
16		E		12	1·50	10·50			3	17·8	0·071	1·33
17		E		12	3·0	9·0			2	18·0	0·083	1·34
18		E		12	4·0	8·0			4	17·7	0·094	1·33
19		E		12	6·0	6·0			3	18·1	0·125	1·35
20		E		12	9·0	3·0			4	20·6	0·250	1·54
21		D_r	¾	12	3·0	9·0	Int.	0·005	3	22·0	0·083	1·31
22		D_r		12	6·0	6·0			2	24·4	0·125	1·45
23		D_r		12	9·0	3·0			2	27·1	0·250	1·61
24		D_r		6	0·75	5·25			2	21·7	0·143	1·29
25		D_r		6	1·50	4·5			2	19·5	0·167	1·16
26		D_r		6	2·0	4·0			2	20·9	0·188	1·24
27		D_r		6	3·0	3·0			2	21·7	0·250	1·29
28		D_r		6	4·5	1·5			2	26·1	0·500	1·55
29	CARPENTER & ROOP	A							9	17·7		1·10
30		B_r							6	18·9		1·27
31		B_n	¾	12	3·0	9·0	Int.	0·005	6	18·3	0·083	1·17
32		C							7	19·5		1·21
33		D_n							7	20·3		1·31
34		E							3	17·1		1·27
35	BOODBERG & COLLABORATORS	A		3	¼	2·5	Ext.		1	27·6	0·300	1·71
36		A		12	3	9·0	Int.		1	17·2	0·083	1·07
37		A		72	18	54·0	Int.		1	16·3	0·014	1·01
38		B_r		3	¼	2·5	Ext.		1	29·5	0·300	1·98
39		B_r		12	3	9·0	Int.		1	19·4	0·083	1·30
40		B_r		72	18	54·0	Int.		1	15·4	0·014	1·03
41		B_n		3	¼	2·5	Ext.		1	28·3	0·300	1·82
42		B_n		12	3	9·0	Int.		1	18·9	0·083	1·21
43		B_n	¾	72	18	54·0	Int.	0·005	1	14·7	0·014	0·94
44		C		3	¼	2·5	Ext.		1	28·4	0·300	1·76
45		C		12	3	9·0	Int.		1	17·8	0·083	1·10
46		C		72	18	54·0	Int.		1	16·6	0·014	1·03
47		H		3	¼	2·5	Ext.		1	30·4	0·300	1·91
48		H		12	3	9·0	Int.		1	21·3	0·083	1·34
49		H		72	18	54·0	Int.		1	17·1	0·014	1·07
50		N		3	¼	2·5	Ext.		1	37·4	0·300	1·45
51		N		12	3	9·0	Int.		1	29·9	0·083	1·16
52		N		72	18	54·0	Int.		1	28·7	0·014	1·11
53	WILSON & COLLABORATORS	E		12	3	9·0			3	17·2	0·083	1·28
54		E		24	6	18·0			3	16·4	0·042	1·22
55		E		48	12	36·0			2	16·0	0·021	1·19
56		E		72	18	54·0			2	14·5	0·014	1·08
57		D_r		12	3	9·0			3	21·3	0·083	1·27
58		D_r	¾	24	6	18·0	Int.	0·005	1	20·5	0·042	1·22
59		D_r		48	12	36·0			1	19·2	0·021	1·14
60		D_r		72	18	54·0			3	18·8	0·014	1·12
61		D_n		12	3	9·0			1	21·3	0·083	1·38
62		D_n		24	6	18·0			1	18·5	0·042	1·19
63		D_n		48	12	36·0			2	17·7	0·021	1·14
64		D_n		72	18	54·0			3	17·0	0·014	1·09
		TOTAL							177			

notched cross section, with material brought to the yield point both in tension and compression. Fortunately, Bagsar's notches of 3/16 in. depth were shallow compared with the notched widths of 4 in. and 6 in., so that the plastic stress distribution at general yield could not be much modified by the notch. A fairly accurate determination of the load just necessary to create the plastic hinge may therefore be made. The stress distribution across the notched zone can be assumed to be as in Fig. 3, with a neutral axis, separating tension and compression zones at yield point stress, whose position depends on the ratio e/d. If the notch is shallow, the load just sufficient to form a plastic hinge can easily be determined by resolving in the direction of tension, and taking moments, as:

$$\frac{\text{P}}{\text{(Notched area of cross section} \times \text{yield point)}} = \sqrt{1+\left(\frac{2e}{d}\right)^2} - \frac{2e}{d} \quad \ldots \quad (2)$$

This is plotted against $\frac{e}{d}$ in Fig. 3. The measured breaking loads for four steels, whose properties are given in Table I, are also shown in Fig. 3, for cleavage-tear tests both parallel and perpendicular to the direction of rolling. It is noteworthy that these measured strengths are consistently just above the full plasticity line, which is valuable proof that plasticity occurs over the complete area, prior to fracture.

Another eccentrically loaded notched specimen used in brittle fracture studies is due to Dr. Tipper[35] and is Tee-shaped, as shown in Fig. 4. In this case, the arms are pulled in a direction transverse to the notch, and at sub-transition temperatures brittle fracture is induced along the leg of the Tee. The two general yield zones which meet at the root of the notch and precede brittle fracture are clearly shown in Fig. 4. The specimen was of low-carbon deep-drawing steel, in order to show the yield zones more clearly. The load at the first sudden appearance of the zones was such that a restraint factor of 0·89 was indicated, when based on the full cross sectional area of the arms. A similar specimen cut from 1¼ in. thick commercial plate (Table I) notch brittle at room temperature, failed by means of a brittle fracture along the leg when the stress applied to the full cross section of each arm was 0·94 times the yield point. This may be compared with the restraint factor of 0·89 mentioned above. Both specimens had sawn notches 0·007 in. wide.

So far, no mention has been made of the effect of notch sharpness. All the specimens described have had artificial notches produced either by sawing, milling or knife pressing, in order of increasing severity. Many brittle fractures in practice are produced from natural cracks due to welding or similar causes, so that specimens with such natural notches ought to be investigated. Robertson[22] has tested three specimens with fatigue cracks, produced by subjecting shallow-notched square tensile bars to 250,000 reversals at a suitable stress amplitude, until the fatigue cracks reached normal notch depth. The results are given in Table IV. Naturally cracked specimens produced by welding (Fig. 5), by means of a technique devised by D. M. Kerr, of Alex. Stephen & Sons Ltd., Glasgow, have been tested by the author. The transverse weld cracks were produced at about 70 deg. C. after laying down submerged-arc welds at the high rate of 1500 amp. on ⅜ in. plate at 18 in. per minute, using carbon backing bars. By this means sufficient carbon was picked up to produce eutectoid weld metal. The cracks occurred regularly at about 6 in. intervals. When opened they had a characteristic blue appearance, compared with the brittle fractured surfaces later produced adjacent to them (Fig. 6). The specimens containing the cracked welds were tested in tension in the direction of the weld after they had been cut to a reduced width. Residual stresses should thereby have been removed. The collected results are given in Table IV.

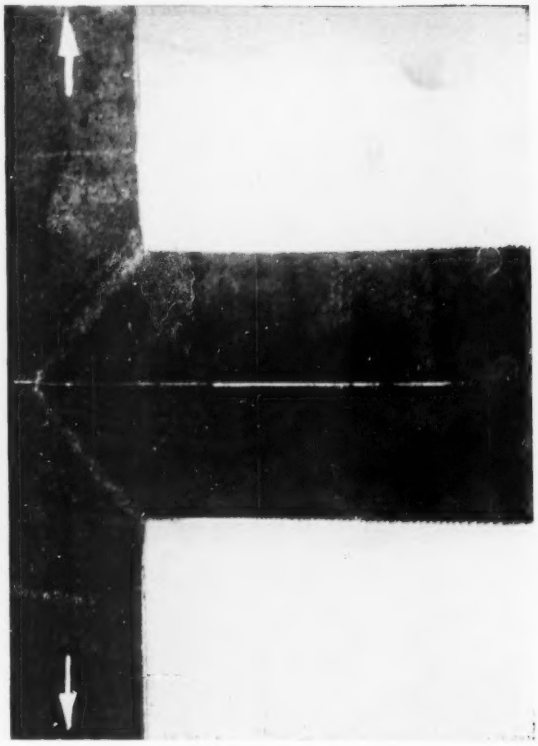


Fig. 4.—General yield of notched tee-shaped tensile specimen.

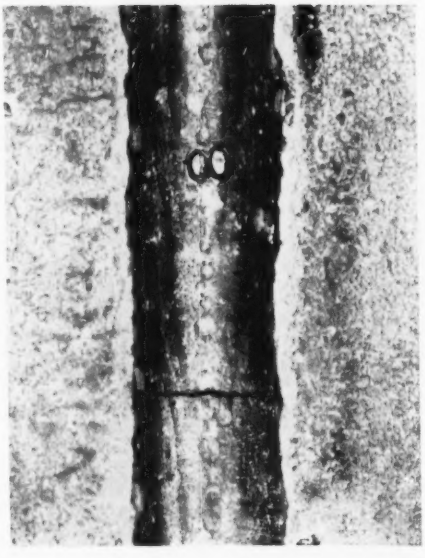


Fig. 5. Transverse crack produced under control in automatic-arc butt weld.

Table IV.—Collected results of tensile tests of naturally cracked steel plates.

Steel	Yield Point (tons per sq.in.)	Testing temp. (deg. C.)	Types of Fracture	Thickness (in.)	Width (in.)	Total Notch Width (in.)	Net Breaking Stress ÷ Yield Point	Calculated Restraint Factor
Robertson: Commercial M.S. bar	15·7	20	Brittle	1	1	0·25	1·27	1·13
Robertson: 1¼in. ship quality plate	16·4	7	,,	1	1	0·25	1·40	1·13
Robertson: 2 in. stiffener: D.W. steel	30·0	+12	,,	1	1	0·25	0·88	1·13
Wells .. Ship quality steel ..	19·8	−2	Ductile	0·37	2·25	0·80	1·38	1·20
Wells .. Ship quality steel ..	19·8	−24	Brittle	0·37	2·25	1·20	1·58	1·41

Table V.—Collected results of full scale tensile tests on ship structures.

Investigator	Type of Structure	Test temp. (deg. C.)	No. of Tests	Average Nominal Cross Section Area (sq.in.)	Average Breaking Strength ÷ Yield Point
De Garmo[23]	Hatch Corner	+21	21	65·3	0·89
Campbell[24]	Londitudinal intersection	−18	4	20·1	1·05
Campbell[25] and collaborators	Bulkhead intersection	−18	4	33·6	0·78

Now these results, with the one exception for the 30 tons per sq.in. yield point steel, show the net breaking stress divided by the yield point to be just greater than the plastic restraint factor, so that it appears that the natural cracks, however produced, are no more severe than the knife-cut notches used by Bagsar (Fig. 2).

Brief mention should also be made of a series of three sets of full-scale static tensile tests, carried out for the Ship Structure Committee of the U.S.A., on elements of ship structure at temperatures low enough to cause brittle fracture. These structural elements were fabricated under normal conditions by welding from ship quality plate of 15·9 tons per sq.in. yield point, and their geometrical complexity indicated that notch effects should be present. A summary of the results is presented in Table V.

The load deformation curves and energy absorption measurements associated with these tests verified that in all cases general yield occurred before failure. Although it is inadequate to compress the results of such extensive tests into a small table, the correlation between the breaking stresses on the nominal cross-sectional area and the yield points of the materials is quite remarkable when the complexity of the structures is considered.

In summing up, it can be stated that the tests of notched shapes, cut from prime plate showing usual ductility in the tensile test, have one thing in common. When tested at temperatures so that notch brittle fractures occur, failure does not take place until loads sufficient to cause general yield are applied, and the general yield zone embraces the root of the notch. General yield is defined as that state in which no path through the plate between two opposite external loading points can exist wholly through elastic, or unyielded, material.

In case it should be objected that, in the cleavage tear test, there is no evidence that the compression and tension zones actually meet before failure, it should be mentioned that the situation in which the plastic zones are actually separated by a small area of unyielded material, is one of instability. In the case of pure plastic bending of a rectangular-section bar, Heyman[26] has shown, by means of a full relaxation analysis, that the end moment is only 7 per cent. less when the plastic zones cover half the depth of the bar, than when they meet and the full plastic moment is achieved. This result confirms the simple theory of plastic bending, for which the fractional reduction of bending moment is given by $\frac{\delta^2}{3}$, *where* δ *is the fraction of depth still elastic. The same state of instability exists in the externally notched bars for which solutions have been given, and where plastic zones from the notch roots coalesce at general yield.*

The Mechanism of Initiation

It is seldom disputed that, even at low external loads, local plasticity must exist at the roots of notches in ductile materials. It is also well known that the true stress on the reduced area at fracture of a mild steel tensile test piece is in the neighbourhood of four times the yield stress, and yet a "silky" ductile type of fracture always occurs. The cleavage strength of the plastically deformed crystal aggregate must therefore be greater than four times the yield stress. If the Ludwik theory is correct, then the stress at the root of the notch is apparently increased in some manner over and above the yield point so that brittle fracture can be initiated.

Several theories have been advanced in explanation of this fact. Elastic superstressing is one of these. The basis of this theory is the observation that the yield point is considerably raised in small bodies of material. If this were to control brittle fracture, however, it is difficult to see how there could be any strength at all in notched bars at sub-transition temperatures, since in a purely elastic material stresses of high value can exist at low external loads. The suppression of plastic flow by triaxial stresses at the roots of notches must also be considered, but for non-strain-hardening materials, the plastic theories outlined above preclude the existence of tensile stresses greater than about 3·5 times the yield point as a result of this suppression. In most of the practical cases, where the initial notch length is a small fraction of the net width, the plasticity studies show that triaxiality hardly exists at all, even for plane strain. Ludwik himself seems to have favoured the increase in yield point as a result of increased strain rate as another explanation. Taylor and Whiffen[27] have shown that threefold increases of yield point in mild steels occur when the strain rate is raised from that in the common tensile test to the order of 10^4 per second. With the known high rates of propagation of brittle fractures, strain rates are likely to be much higher than this. Taking a final strain of unity, occurring over 0·1 in. of wave travel, a strain rate of the

order of 10^6 per second results. This explanation, then, is acceptable for the conditions obtaining during propagation, but not during initiation.

Strain hardening is one remaining means by which the tensile stress at the root of the notch could increase to a value compatible with the initiation of cleavage failure. Boodberg and collaborators[7] have shown conclusively, by means of micro-hardness surveys, that strain-hardening accompanies brittle fracture. One of their conclusions reads: "Extensive deformation occurs at the base of the notch for both shear and cleavage fractures. The strain hardening which occurs in this region is as great as that which occurs near the point of fracture in a standard tensile test bar. It appears that more plastic flow occurs near the surfaces of the plate than at points remote from these surfaces." Their results also strongly indicate that, for brittle fractures, the strain hardening is concentrated at the roots of the original notches, whereas for ductile fractures it is uniformly distributed over the fracture surface. This bears out the suggestion that the increase of yield point due to high strain rate may be sufficient to maintain a high-velocity brittle fracture when once initiated.

The strongest confirmation for the strain-hardening theory comes from the foregoing generalisation of experimental observation that an external load high enough to cause general yield, with the general yield zone embracing the root of the notch, is necessary for initiation to take place. Before general yield occurs, the loading points are joined by a continuous length of purely elastic material, so that the change in relative position of these points of application is small. It therefore follows that general yield should occur before any large plastic strains become possible, at the roots of notches or elsewhere.

In order to check the validity of this hypothesis, plastic strains have been measured on an axially loaded, externally notched plate specimen of low-carbon deep-drawing steel of $\frac{1}{8}$ in. thickness. This steel was chosen because of its sharp yield point and low strain hardening rate. The dimensions of the specimen and positions of strain measuring points are shown in Fig. 7. Strain in the axial direction, on a line precisely through the root of one of the sawcut notches of 0·004 in. width, was measured by means of a specially constructed extensometer of 0·038 in. gauge length. A single-lever magnification in this extensometer proved to be adequate, because strains of the order of unity were to be measured. Huggenberger extensometers of 1 in. gauge length were set simultaneously to indicate longitudinal and transverse strains at the centre of the specimen. The stress/strain curves are shown, together with that for the material itself, in Fig. 7. It should be emphasised that, in this diagram, the strain scale for the root of the notch has been diminished by the factor 200 for comparison with the other measured strains. The long-base strains at the centre of the specimen have values of initial slope different from the equal elastic values predicted by the Neuber elastic theory, and also show perturbations due to the propagation of the plastic zones, but these do not obscure the remarkable general similarity with the stress strain curve for the root of the notch. The marked increase of strain at the root of the notch when general yield takes place is clearly shown. It is interesting to compare the strain concentration factor at the notch, which appears to be about 200, with the elastic stress concentration factor of 28·5 which would obtain for a notch of this acuity when calculated from the Neuber theory.[12]

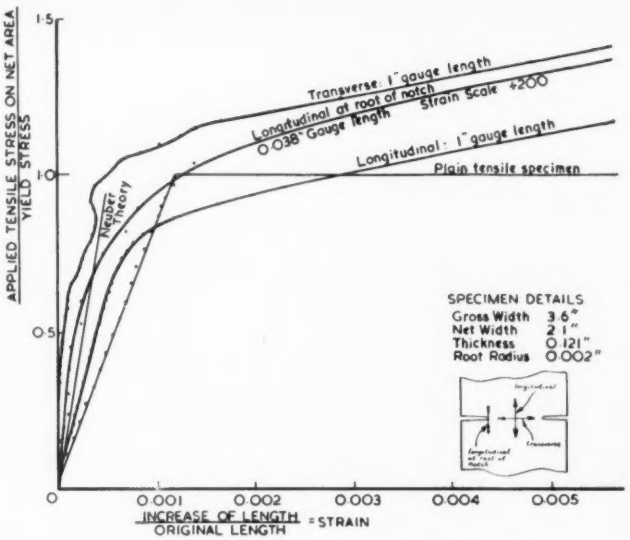


Fig. 7. Strain measurement on externally notched plate.

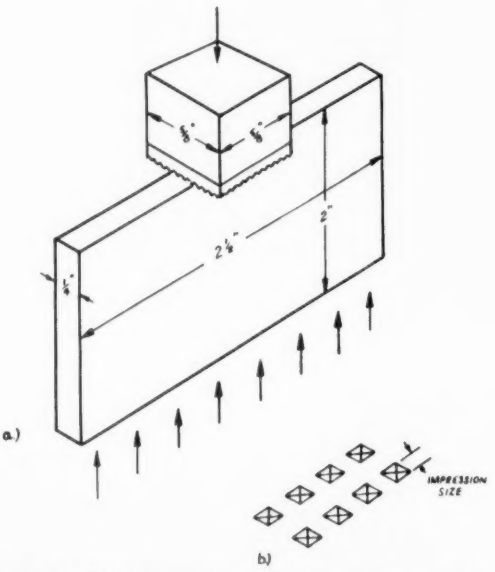


Fig. 8. Rigid die experiment. General arrangement and method of impression size measurement.

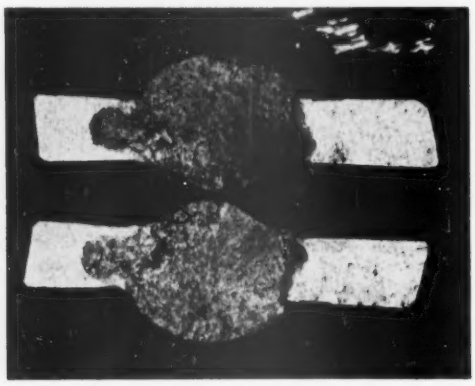
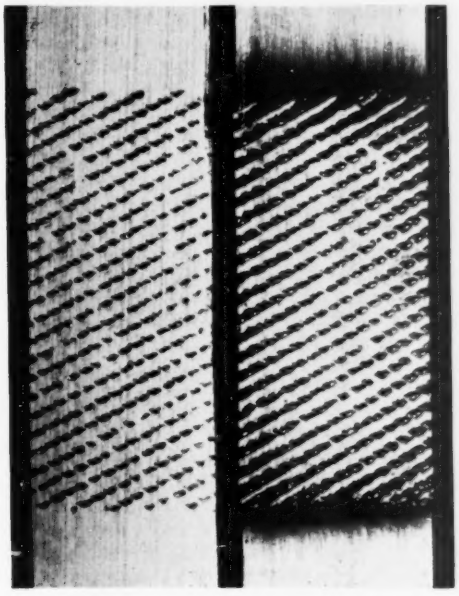


Fig. 6. Brittle fracture produced under static tension from cracked butt weld of type of Plate 2.

A graphical demonstration of the strain-hardening type of concentration of stress at the root of a notch may be made by means of a very simple experiment with a plate and hardened flat die, arranged as in Fig. 8. Hencky[28] seems to have been the first to show that this case is identical with the externally notched tensile case, except that the signs of the loads and strains are reversed. The face of the die consists of a section from a second cut flat file, so that a plane contact area exists, having diamond-shaped teeth distributed over it, uniformly as to spacing and height. When loaded, as in Fig. 8, the die remains flat and rigid while the plate deforms plastically. While the load averaged over the indented area (plate thickness × die width) is about equal to the yield point of the material of the plate, and general yield is just prevented, the tooth impressions are of uniform size over the total area of impression (Fig. 9a), indicating almost uniform distribution of contact pressure. At over double the load the concentration of pressure towards the ends of the die is seen by the heavier tooth impressions existing there. In this particular case the concentrated pressure has even outweighed the increased resistance to individual tooth indentation by local strain hardening.



(*a*) (*b*)

Fig. 9. Rigid die impression, showing effect of strain hardening.

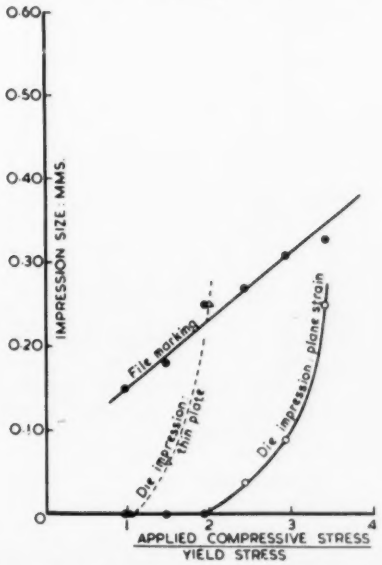


Fig. 10. Rigid die experiment. Impression size calibration for pressure estimation.

These tests were extended quantitatively by calibrating contact pressure against tooth impression size measurements, as in Fig. 8b, and made by micrometer microscope. In order to secure an adequate uniformity of contact pressure during calibration, the die was pressed against the flat face of a large block of mild steel, instead of the plate of Fig. 8a. In this case, from the Prandtl-Hencky-Hill plasticity solution given above, general yield was known not to occur until an average pressure 2·57 times the yield point was reached, so that uniform contact pressure could be expected up to this applied average pressure. The calibration is given in Fig. 10, together with measurements of the total die sinkage into the plate. The measurements of contact pressure distribution in the experiments of the aforementioned type, and obtained from the calibration, are shown in Figs. 11, 12 and 13. These measurements represent the mean contact pressures over the whole thickness at each position along the plate under the die. The loads applied were equivalent to mean pressures equal to the yield point and $2\frac{1}{2}$ times this value in each case, as shown by the horizontal lines. The measured pressures for these two load levels are shown as follows: dots—yield point; triangles—$2\frac{1}{2}\times$ yield point. Fig. 11 shows the situation where the equivalent of a notch-root radius of 0·015 in. is present. As the load is raised it is seen that the uniform pressure distribution changes to one in which there is a concentration of pressure at the root. In Fig. 12, which illustrates the case where there is no root radius, the concentration of stress at the root seems to have begun, even at an applied average stress equal to the yield point. In Fig. 13, where the plate width is restricted, as in the case of shallow notches, the uniform pressure distribution at the lower load again appears. In every case the area under the measured pressure distribution curve is sensibly equal to that under the applied average pressure level, so that the impression calibration is vindicated.

The strain hardening theory for initiation of notch brittle fractures under static loads is thus supported by three independent methods of experiment. Firstly, the existence of pronounced strain hardening at the roots of notches in tensile plates, in which brittle fractures have actually occurred, is shown by micro-hardness surveys. Secondly, in a weakly strain hardening material, a rapid rise of strain occurs at the root of a notch when general yield occurs and embraces the root of the notch. Thirdly, in a die experiment, the incidence of concentrated stresses at the root is seen, in some cases, to coincide with an increase of load beyond that necessary to cause general yield embracing the root. As extreme strain hardening only occurs after the onset of general yield embracing the root of the notch (as shown by conditions two and three), and notch brittle fracture only follows after the same plastic condition has been reached, these results can be regarded as conclusive in favouring this theory.

Implications of the Strain Hardening Theory

It is legitimate criticism that the plasticity and strain hardening theory of initiation, in its present form, fails to show why it is that a brittle fracture can occur at a relatively shallow notch and yet ductile fracture always occurs in a normal tensile specimen, when the triaxiality appears to be both similar and small in both cases, assuming that the

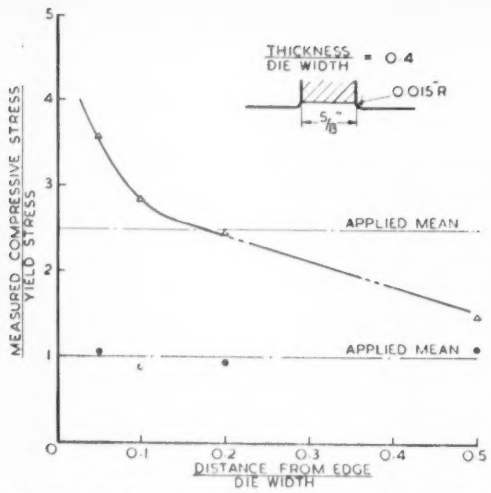


Fig. 11. Rigid die experiment. Pressure distribution with large root radius.

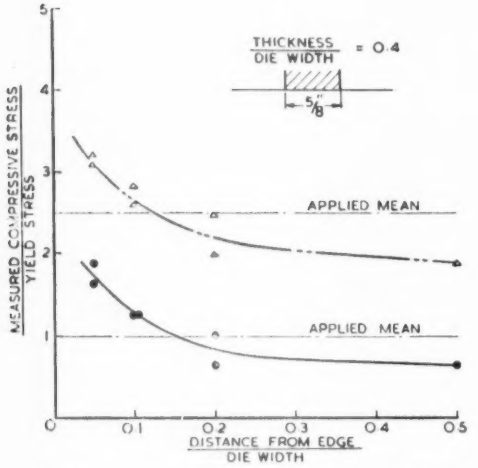


Fig. 12. Pressure distribution with no root radius.

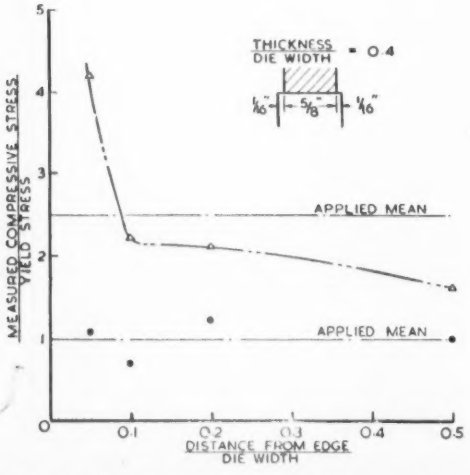
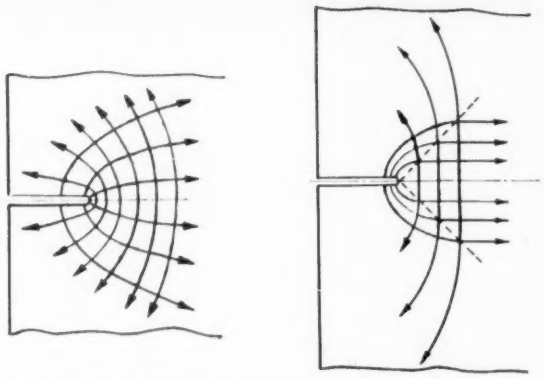


Fig. 13. Pressure distribution with shallow notches.



(*a*) With strain hardening. (*b*) Without strain hardening.

Fig. 14. Stress trajectories with and without strain hardening.

plasticity solutions are accepted for the former case, and an analysis such as that of Bridgman[29] is accepted for the latter. For a quantitative answer, a plastic theory embracing strain hardening must be awaited, but, qualitatively, a simple explanation may be given: this is that the effect of strain hardening at the root of a notch is to increase triaxiality, by virtue of the increasing concentration of stress at the root of the notch to which it gives rise. Physically, this may be proved by a simple consideration of equilibrium. When a concentration of longitudinal stress occurs at the root, the trajectories of principal stress are crowded at the root so as to take on a high curvature, as in Fig. 14a. In this case equilibrium can only be maintained by generating a high transverse stress of the same sign. In the simplest plastic case, without strain hardening, stress is thrown off at the root, and no crowding, or high curvature of the trajectories occurs (Fig. 14b). The process may be seen to be gradually taking place in the curves of transverse and longitudinal strain against load in the centre of the externally notched bar of Fig. 7. At the commencement of loading until an average net stress of 0·3 × yield point has been reached, the ratio of the former to the latter strain is zero. When the net average stress has exceeded the yield point the ratio has already reached 0·2, and the parallel slopes of the two curves suggest that a transverse to longitudinal strain ratio of unity might easily be achieved after further strain. It is an accepted axiom in plasticity that strain rates at a point are proportional to the deviator stresses.* Equal strain rates of the same sign in the transverse and longitudinal directions therefore suggest both equal deviator and total stresses in these two directions. It should be emphasised, however, that triaxiality of this type in a material exhibiting plasticity cannot occur before general yield has taken place.

In view of the foregoing conclusion with regard to the necessity for strain hardening to take place at the root of a notch before a brittle fracture can be initiated by static load, in a material showing usual ductility in the tensile test, it is interesting to re-examine the results of Fig. 2. The experiments of Bagsar on the one hand, and Boodberg and collaborators on the other, are noteworthy as showing low and high breaking strengths, respectively, at a thickness/width ratio of about 0·3. Both sets of results involve specimens 3 in. wide and $\frac{3}{4}$ in. to 1 in. thick, with notches 3/16 in. to $\frac{1}{4}$ in. deep. The main difference is that Bagsar's notches were knife-pressed to a root radius of 0·0015 in., whereas those of Boodberg and collaborators were sawn by means of a jeweller's saw blade 0·01 in. thick. Apart from the difference

* Mean stress + deviator stress = Total stress.

of root radius, the extremes of strain hardening which these two methods of manufacture would produce, must undoubtedly have had considerable influence. The difference in root radius could more simply offer an explanation, and yet specimens of similar dimensions, shown in Fig. 2 as due to Dr. Tipper, but with notches milled to 0·01 in. root radius, show breaking stresses between those obtained by the two other investigators, and the naturally cracked specimens of vanishingly small root radius appear to be no weaker than those with knife-pressed notches. The difference in strain hardening produced by the methods of machining is certainly the more plausible explanation, particularly as milling (large teeth) is clearly between sawing (small teeth) and knife-pressing, in the severity of strain hardening produced adjacent to the cut.

THE PROPAGATION OF NOTCH BRITTLE FRACTURES

So far, it has only been ascertained that notch brittle fractures are initiated under static loading by the same mechanism as ductile fractures. This may even appear to be obvious, since it is well known that there is often no appreciable transition between the static strengths of sharply-notched tensile specimens when the mode of failure changes from ductile to brittle by virtue of a temperature change. Any limited change of breaking load may be ascribed to the greater amount of strain hardening which takes place in the ductile mode in the body of the plate, because of the additional total extension which must be suffered. There is therefore at least a second condition to fulfil if a brittle fracture, once initiated, is to propagate. This condition could still be that suggested by Irwin[10] and mentioned in the Introduction, namely, that the energy to be fed into the advancing crack should be adequately provided by the elastic strain energy stored in the plate without recourse to the energy provided by external loads, which is evidently necessary for ductile fractures to propagate. In order to check this, it therefore becomes necessary to attempt to measure the surface energy requirement of a propagating brittle fracture.

The Measurement of Surface Energy

Although the presence of plastic flow at the surface of a brittle fracture may be detected by the line broadening to which it gives rise in X-ray diffraction photographs,[30] the determination of the amount of flow, and its distribution in depth, is a matter of some difficulty. Since the total dissipation of energy on unit surface is required, without the necessity for reference to stress and strain, the relation between plastic work and heat evolution would seem to be useful. Taylor and Quinney[31] have demonstrated that the conversion factor for plastic work to heat evolution is of the order of 0·90, the remainder of the energy of plastic flow being stored as potential energy of strain in the crystal lattice. Further, the enormous speed of a brittle fracture, when compared with a temperature wave, would enable the system accurately to be treated as an instantaneous plane source giving pure one-dimensional heat flow. The temperature rise method of measurement was thus chosen.

Assuming the source strength to be q heat units per unit area, produced instantaneously, Carslaw and Jaeger[32] show the temperature rise T, above that of the surroundings, at any time t after release, at distance x from the plane of the source, to be

$$T = \frac{qe^{\frac{-x^2}{4\alpha t}}}{2\rho c\sqrt{\pi\alpha t}} \quad .. \quad .. \quad .. \quad .. \quad (3)$$

where α is the thermal diffusivity of the material (related to the thermal conductivity, k, as follows: $\alpha = k/\rho c$), ρ is the density, and c the specific heat, respectively.

The maximum temperature reached, at distance x, is

$$T_{max} = \frac{q}{2\rho cx}\sqrt{\frac{2}{\pi e}} \quad .. \quad .. \quad .. \quad (4)$$

and occurs after a time t_o, where

$$t_o = x^2/2\alpha \quad .. \quad .. \quad .. \quad .. \quad (5)$$

The above expression for T_{max} may be rewritten, if T_{max} is measured and q is required, as

$$q = \sqrt{2\pi e}\ \rho cxT_{max} \quad .. \quad .. \quad .. \quad (6)$$

For steel, in which $\rho = 7{\cdot}86$ gm. per ml., $c = 0{\cdot}11$ cal. per gm. deg. C., and x is in cm., this equation for the surface energy S becomes

$$S = 71{\cdot}8\ xT_{max} \text{ ft. lb. per sq.in.} \quad .. \quad .. \quad (7)$$

Thus, a surface energy of 10 ft. lb. per sq.in. causes a maximum temperature rise of $\frac{1}{2}$ deg. C. at a distance from the fracture of 2·8 mm. This temperature is reached only after a time lapse of 0·48 sec., which is sufficiently slow to avoid confusion with adiabatic cooling of the material, following release of elastic stress. This method of measurement would therefore seem to allow high accuracy, since there is no dependence on physical constants other than those accurately measurable.

In view of the convenient times and temperature rise values to be expected, a simple thermocouple and mirror galvanometer system, in conjunction with a recording camera, have been employed for all measurements made so far. The thermocouple system has consisted of pairs of 45 per cent. nickel/copper wire, soft soldered to each specimen, as shown in Fig. 15, to make the specimen part of the circuit. The relative positions of the hot and cold junctions are approximately 0·1 in. and 1 in. from the expected fracture line, and on the same side of it, to avoid break of circuit at fracture. The temperature wave can thus always be assumed to be strongly attenuated when reaching the cold junction. The thermocouples were attached to a D'arsonval type galvanometer, having a periodic time of 0·1 sec., sensitivity 100 mm. per micro-amp., at 1 metre, coil resistance 300 ohms, and external resistance for critical damping of 4000 ohms. The instrument was thus somewhat overdamped,

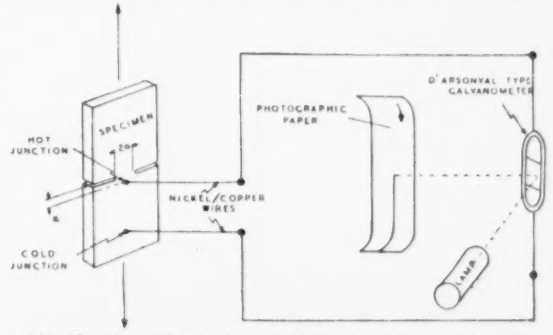
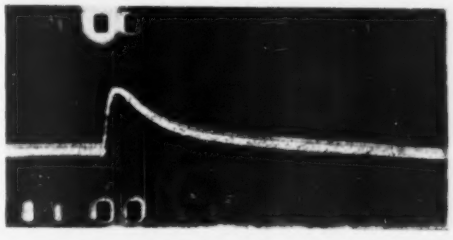


Fig. 15. General arrangement of apparatus for temperature wave recording.



time →

Fig. 16. Temperature wave record.

but this could be tolerated in view of the margin between the natural frequency and that to be applied as a result of the temperature wave. Photographic recording was used, with a paper speed of about 1 cm. per sec. A static calibration was employed to determine the temperature sensitivity. This was found to be 0·45 cm. per deg. C., for the thermocouple. One of the recorded temperature waves is shown in Fig. 16.

A further development has been to measure the thermocouple voltage from the temperature wave as an integral with respect to time to give a pointer reading, and so to dispense with the recording camera. For this purpose a number of simple and sensitive flux meters have been constructed. A description of these instruments and their calibration will be published later.

When the investigation began, no commercial mild steel was available which was sufficiently notch brittle at room temperatures for surface energies to be conveniently measured. Flat sections, 1¾ in. by 3/16 in., of higher-carbon automobile spring quality steel (see Table I) were therefore investigated in the externally notched condition. Fifteen deg. notches with a root radius of 0·007 in. were cut on the earlier specimens. Later it was found that sawn notches, finished with a jeweller's saw to a width of 0·008 in., were equally effective. The more deeply notched specimens were loaded through pins, to avoid eccentricity; the others were held in wedge grips in the tensile testing machine. The measured breaking strengths and surface energies are shown in Table VI, for twenty-four specimens notched to net widths from 1½ in. to 1/16 in. Some of these specimens gave partially ductile failure so that large surface energies resulted. For the cases where the net widths 2a (Fig. 1a) were comparable with the distances x from the thermocouple to the fracture plane, corrections were applied to T_{max}, in equation (7), in order to compensate for the curvilinear, rather than plane, heat flow in such a case. This correction factor was found, from heat flow equations (see Appendix 1), to be

$$\text{Correction factor} = \sqrt{\frac{\pi}{32e}}\,\frac{(2m^2+1)^2}{m} \text{ where } m = \frac{x}{a} > 1 \quad (8)$$

Measured surface energies are plotted in Fig. 17 against the fractions of the bar widths occupied by notches. A noticeable feature of this curve is the tendency for the surface energy to approach a minimum value of approximately 16 ft. lb. per sq.in. for the bars notched to less than about half the original width. Furthermore, there is a rough correlation between surface energy and appearance of fracture. The high values of surface energy correspond to ductile fractures, the intermediate values to a rough type of brittle fracture, and the base values to a very clean brittle fracture. The guess may be made that the correlation extends to velocity of fracture, the basic surface energy corresponding to the limiting high velocity which Mott[33] has suggested should approach a definite fraction of the speed of sound in the material.

Some results of M. de Leiris[38] are interesting in this context. He pulled at a range of temperatures a number of mild steel bars 1⅛ in. × 1½ in. with pairs of standard Izod Vee-notches 0·118 in. deep. Some of these bars were of 10·75 in. gauge length, while others were 13 ft. 9 in. gauge length, and the net average breaking strengths were 33·2 tons per sq.in. in the former case, and 28·1 tons per sq.in. in the latter. The difference was established as being statistically significant and, while the 7·5 fold difference in strain rate could not be ignored, de Leiris concluded that the difference in amount of elastically stored energy was the more likely cause. Now, these shallow notched bars would behave similarly to their spring steel counterparts in the experiments just described, and would be likely to show large surface energies. In fact, the fractures were described as partly ductile by de Leiris himself. Under these conditions of possibly slow, and only partly brittle fracture, the elastic energy stored in the bars remote from the notched zone would undoubtedly be usable, whereas it would obviously not be so for a fast fracture, of velocity approaching three-quarters of the speed of elastic shear waves in the material.

Comparison with Griffith-Irwin Theory

It was realised at an early stage that the experiments would be expedited by the use of externally rather than internally notched bars. The Griffith formulae were therefore worked out for the deep and shallow external notch cases (see Appendix 2), using elasticity solutions given by Neuber.[12] These formulae are, for plane stress:

$$\text{Deep external notches } \sigma = \sqrt{\frac{\pi ES}{4a}} \text{ or } P = \sqrt{\pi ESa} \quad .. \quad (9)$$

$$\text{Shallow external notch } \sigma = \sqrt{\frac{3ES}{2a}} \quad .. \quad .. \quad .. \quad (10)$$

In these cases σ is the average breaking stress on the net width $2a$, while P is the breaking load per unit thickness. E is Young's modulus and S the surface energy, in consistent units. A further useful surface energy formula for a disc-shaped internal crack of radius a in an infinite body, is given by Sack[34] as

$$\sigma = \sqrt{\frac{\pi ES}{4a(1-\nu^2)}} \quad .. \quad .. \quad .. \quad (11)$$

ν is Poisson's ratio. All these surface energy formulae are transposed into convenient engineering units in Appendix 2.

In Fig. 18, a comparison between the measured surface energies and the breaking loads, both shown in Table VI, is made possible by plotting P against $\sqrt{2aS}$. The linear relationship suggested by equation (9), with the slope $\sqrt{\frac{\pi E}{2}}$ is strikingly borne out, when the measured value of E = 12,500 tons per sq.in. is used. Within the limits of error for the experiment, the adherence to the theoretical line is reasonable, with the exception of those specimens showing partly ductile fractures. For ductile fractures, a considerable contribution from the external loads towards the energy of fracture would be anticipated in any case.

In view of the foregoing conclusion that the existence of fully plastic zones must precede fracture by means of

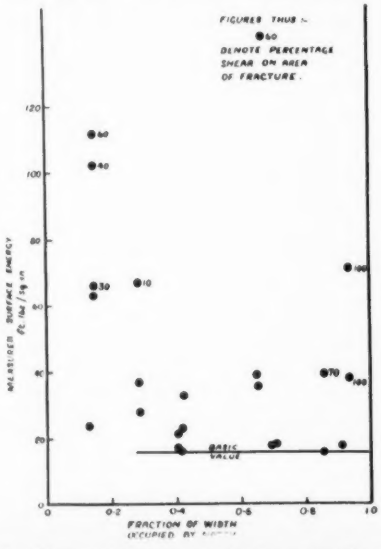


Fig. 17. Measured surface energies for externally notched spring steel plates.

externally applied static loads, the exact correlation of Fig. 18, whose co-ordinates are based on elastic theory, may well be considered surprising. However, it should be remembered that, no matter how much plastic deformation occurs, a specimen may always be unloaded elastically at any moment before final failure takes place. Although the result is not shown in Fig. 7, in order to avoid confusion this was clearly demonstrated to be true with the externally notched bar used in this experiment.

A difficulty which might well have interfered with the correlation shown in Fig. 18 is the measurement of the surface energy at the centre of the specimen, rather than near the notch root, as demanded by the theory. This was necessitated experimentally because of the assumption of plane heat flow, which would only obtain at positions remote from the ends of fractures. For the time being, it can only be assumed that the propagating brittle fractures which were examined persisted at least for short distances at the surface energy levels at which they commenced. Further tests with multiple measurements on larger specimens would help to elucidate this point.

In Fig. 19 the data of Table VI are plotted in terms of the average net tensile stresses at failure and the fraction of the total width occupied by the notches. Firstly, it is seen that these stresses are all above the yield point, and even greater than 2·6 times this value for the very deeply notched specimens, which might be identified with plane strain. The initiation condition is therefore seen to be satisfied. Further, for the conditions where the bars are notched to leave a ligament less than about 0·2 of the total width, the fractures occur at loads closely predicted by equation (9), using the basic value of S = 16 ft. lb. per sq.in., determined independently by the temperature rise measurements. This good agreement would not be expected for some of the less deeply notched bars, because of the higher surface energies experienced (Fig. 17) and also because of the appearance of shear in the fractures. Equation (10), rather than (9), would be more correctly used in this range, but the error in adhering to equation (9) does not seem to be large.

For a true brittle fracture, in which evidence of ductility is absent, the experimental evidence suggests that the energy for propagation can come entirely from the release of elastic strain energy in the plate. Furthermore, it now also appears that, under the same truly brittle conditions, as mentioned above, the exact breaking load may be predicted from a knowledge of the basic surface energy of the material, as long as this breaking load is known to be well above that necessary for initiation. The brittle initiation condition does not seem to have been completely satisfied in the case of the shallow notched specimens of Table VI and Figs. 17 and 19, as is seen by the high surface energies and breaking loads which were measured. This fits remarkably well with the observation in the section on Initiation that the generation of high plastic stresses may not easily be achieved with the lack of triaxiality consistent with shallow notches, and is even more difficult to achieve with plane stress.

The measured high surface energies, which are not definitely associated with ductile fractures, are of great interest because they may well be symptomatic of the behaviour of the much larger notched specimens in which the load at failure is controlled entirely by the initiation condition. Fig. 18 suggests that there is equality of elastic energy given up, with surface energy absorbed, even where the measured surface energies are above the basic value, as is the case for most of the points in the upper two-thirds of the diagram.

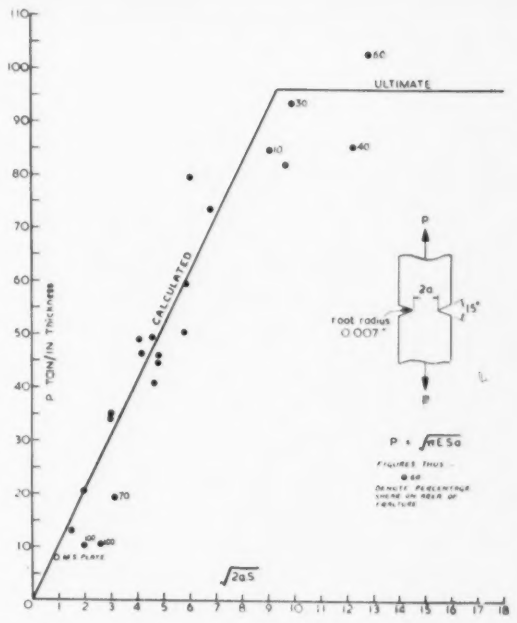


Fig. 18. Measured and calculated surface energies for externally notched spring steel plates.

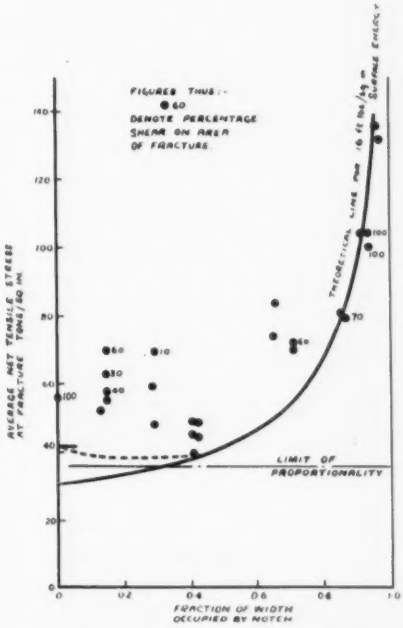

FIGURES THUS :-
60
DENOTE PERCENTAGE SHEAR ON AREA OF FRACTURE
THEORETICAL LINE FOR 16 ft lbs/sq in SURFACE ENERGY
LIMIT OF PROPORTIONALITY
AVERAGE NET TENSILE STRESS AT FRACTURE TONS/SQ IN
FRACTION OF WIDTH OCCUPIED BY NOTCH

Fig. 19. Measured breaking strengths for externally notched spring steel plates.

Surface Energy in Mild Steels

The results of Table III may be used to calculate surface energies of propagation relevant to the mild steels used in these particular experiments. Only the minimum values are of interest, but these have been abstracted in Table VII, and two further results added, for three different types of notched bar.

The experiments of Robertson require qualification, because the initiation of the crack in material with a local temperature as low as −60 deg. C. was produced by a transverse high-speed mechanical hammer blow while the specimen was under the static stress indicated. The results relate to three specimens from 1 in. and $\frac{3}{8}$ in. mild steel plates in which

Table VI. Measurement of surface energy and break loading on externally notched spring steel plates.

	Net Width at Notch $2a$ (in.)	Thickness (in.)	Breaking Load per in. Thickness P (tons per in.)	Net Average Breaking Stress (tons per sq.in.)	Percentage Ductility on Fracture Zone	Type of Grips	Thermocouple Distance x (cm.)	Maximum Temperature Rise (deg. C.)	Surface Energy S (ft. lb. per sq.in.)	$\sqrt{2aS}$
1	1·73	0·151	96·1	55·5	100	Flat Wedge	—	—	—	—
2	1·50	0·144	79·3	52·8	—	,,	0·30	1·11	23·9	5·99
3	1·47	0·170	85·3	58·0	40	,,	0·40	3·55	102·0	12·25
4	1·47	0·167	102·8	70·0	60	,,	0·35	4·44	111·7	12·82
5	1·48	0·160	93·3	63·0	30	,,	0·23	4·00	66·1	9·90
6	1·48	0·166	81·9	55·3	—	,,	0·33	2·67	63·3	9·67
7	1·24	0·154	73·4	59·2	—	,,	0·27	1·91	37·0	6·77
8	1·22	0·174	84·8	69·5	10	,,	0·35	2·67	67·2	9·06
9	1·23	0·152	59·6	48·4	—	,,	0·28	1·40	28·1	5·88
10	1·00	0·141	48·9	48·9	—	,,	0·21	1·11	16·7	4·08
11	1·02	0·151	50·3	49·3	—	,,	0·43	1·07	33·0	5·80
12	1·00	0·150	44·4	—	—	,,	0·26	1·24	23·1	4·80
13	1·02	0·152	46·2	45·3	—	,,	0·41	0·58	17·0	4·17
14	1·02	0·146	40·7	39·9	—	,,	0·31	0·95	21·7	4·65
15	0·48	0·185	34·7[1]	72·3[1]	—/50[1]	,,	0·27	0·95	18·6	2·98
16	0·50	0·198	34·9[1]	69·8[1]	—	,,	0·27	0·96	18·1	3·01
17	0·61	0·195	45·3	74·3	—	Pinned	0·40	1·35	39·0	4·88
18	0·59	0·177	48·5	82·2	—	,,	0·17	2·93	35·7	4·59
19	0·25	0·197	20·2	81·1	—	,,	0·16	1·36	15·6	1·98
20	0·235	0·179	18·5	78·7	70	,,	0·19	2·89	39·4	3·04
21	0·130	0·200	13·6	104·4	—	,,	0·22	1·13[2]	17·8	1·52
22	0·100	0·179	10·45	104·5	100	,,	0·20	4·95[2]	71·0	2·66
23	0·102	0·175	10·22	100·1	100	,,	0·16	3·36[2]	38·6	1·98
24	0·062	0·172	8·50	136·0	—	,,	—	—	—	—
25	0·050	0·152	6·65	132·0	—	,,	—	—	—	—

[1] Breaking loads redetermined with pinned grips.
[2] Corrected for end effect.

Table VII. Minimum surface energy determinations for notched mild steel plates.

Investigator	Type of Notch	Net Notched Width or Notch Length (in.)	Net Average Breaking Strength (tons per sq.in.)	Equation No.	Calculated Surface Energy (ft. lb. per sq.in.)	Measured Surface Energy (ft. lb. per sq.in.)
Tipper	Deep external	0·75	27·0	9	4·77	—
Wells	,, ,,	0·171	44·7	9	3·25	5·75
Robertson	Shallow external	10·00	7·5	10	2·61	—
Bagsar	,, ,,	2·81	16·1	10	3·38	—
Thomas and Windenburg	Internal	0·375	23·8	1	4·60	—

the crack stopped in material near room temperature. By reducing the static tensile stress to 5 tons per sq.in., short brittle cracks from the notch roots could also be produced in the three materials, but these were stopped in material at temperatures above −35 deg. C.

The close comparison between these results, obtained with widely different specimen types, and each with a different formula for calculation, suggests that the minimum surface energy has a real existence for the mild steels immediately below their transition temperatures, just as for the spring steel described above. In this case, however, the minimum surface energy appears to be about 4·5 ft. lb. per sq.in.

A few direct measurements were made by the temperature wave method of minimum surface energies in externally notched specimens cut from 1¼ in. mild steel plate (Tables I and VII), notch brittle at room temperature. In some of these experiments the thermojunction adjacent to the fracture face was placed in a small drilled hole reaching to the centre of the specimen, while for the others it was placed on the outer surface. Absolutely no significant difference in measured surface energy was apparent. Specimens having a minimum notched width at least as low as 0·171 in. wide at the notched zone were necessary, in order to ensure that the breaking load was controlled by minimum surface energy, and not by initiation. Under these conditions special precautions were essential in order to secure truly axial loading. The reason for this is apparent in that the elastic energy stored in a rectangular tie bar is multiplied by the factor $1 + 3\left(\frac{e}{a}\right)^2$ for eccentricity e of the given force, when the total width is $2a$. The analogy may be extended to the notched section of such a tensile specimen as mentioned above since, when this waist is narrow compared with the gross width, an eccentricity of half the ligament width, quadrupling the energy, is easily possible when unarticulated grips are used in the testing machine. Before using articulated grips, surface energies up to 25 ft. lb. per sq.in. were measured with these specimens at similar breaking loads. However, when adequate precautions were taken to eliminate eccentricity of loading, a measured value of surface energy of 5·43 ft. lb. per sq.in. was obtained, corresponding with a value of 3·25 ft. lb. per sq.in. calculated from the breaking stress. Even this apparently large discrepancy can be explained by the existence of no more than 0·025 in. eccentricity, using the above theory.

The reappearance of small shear "thumbnail" zones at the roots of the notches, was taken to indicate that the minimum surface energy was being reached, since these zones had not been present in less deeply notched specimens of the same steel, nor in those in which no special precautions had been taken to eliminate eccentricity.

The Conception of Minimum Surface Energy

By careful examination of fractured surfaces, Dr. Tipper has shown that, even under the most brittle conditions, only a certain proportion of the crystals in the aggregate fail by cleavage. She attributes this to differences in orientation, and it may easily be imagined that this would be so if the cleavage and shear strengths were comparable in magnitude. The fracture planes are different in these two modes of failure so that, even for identical crystals in the aggregate and a uniform distribution of stress, some would be more favourably disposed for fracture by one mode or the other. The effect of orientation of load with respect to the cleavage plane of zinc and cadmium crystals, in which the shear and cleavage planes are identical, has been studied by Schmid[36] and Georgieff and Schmid[37] respectively, at the low temperatures where shear is inhibited. They show conclusively that cleavage is governed entirely by the component of stress normal to the cleavage plane.

If this is so, and a limiting strain rate is assumed, corresponding with the greatest possible velocity of brittle fracture, the conception of a minimum surface energy has a real physical basis. With random orientations of individual crystals, and given shear and cleavage strengths at a certain temperature for the most favourable orientations, a fixed proportion of the crystals in the fracture surface will follow each mode of failure. If, for instance, equal shear and cleavage strengths are assumed and the shear and cleavage planes are equiangularly spaced, the distribution might be half-and-half, but the exact ratio will obviously be dependent on temperature.

A rough calculation of the value of surface energy under these conditions also assumes the cleavages to take place with no energy absorption, and the shearing crystals to behave like elementary tensile specimens. Assuming each shearing crystal to be a cube of side δ, with a tensile strength (shear mode) of σ, and strain at failure e (neglecting the change of σ as strain is increased), the value of S becomes

$$S = \tfrac{1}{2}\delta e\sigma$$

With $\delta = 0{\cdot}005$ in., $e = 1$, and $\sigma = 30$ tons per sq.in., values which might be typical of a mild steel at a high strain rate, it is found that S becomes of the order of 14 ft. lb. per sq.in. Although this is only a crude calculation, the correct order of magnitude of the result supports the orientation hypothesis. The existences of different crack velocities and surface energies greater than the minimum value are not easily explained at this stage.

The conception of minimum surface energy does, however, give an explanation of the effect of absolute plate thickness on transition temperatures, which is known to exist even where metallurgical changes are excluded, as, for instance, when thick plates are machined to smaller thicknesses. Suppose, for instance, that the minimum notched width for a deeply notched specimen for brittle fracture at a given temperature in 1 in. plate is 3/16 in., giving a surface energy of 3·3 ft. lb. per sq.in. and an average net failing stress of 45 tons per sq.in. or $2\frac{1}{2}$ times the yield strength. If, then, the plate thickness is reduced to $\frac{1}{8}$ in., plane strain will no longer exist and general yield will occur at about half this stress. Ductile failure will then occur considerably before 45 tons per sq.in. is reached.

BRITTLE FRACTURE IN LARGE STRUCTURES

A consistent pattern of behaviour, as controlled by the two conditions of crack initiation and propagation, is seen in the results of the comparatively large number and variety of notched and statically loaded tensile specimens dealt with in the previous sections on Initiation and Propagation. In spite of this the crack initiation condition, in particular, seems at first sight incompatible with the fractures occurring under service conditions, as mentioned in the Introduction. It appears that general yield embracing the root of the notch seems to be needed to break a laboratory specimen under statically applied load, yet full-size structures show notch brittle fracture with static external loads well below values sufficient to cause general yield. However, the existence of flaws such as weld hot cracks or structural or metallurgical notches, at the commencements of service brittle fractures is seldom denied. Further, the surface energy condition of propagation can easily explain the extensive spread of brittle fractures, in the later stages, through material under comparatively low tensile stresses from external loads.

It is sometimes said that the conditions for initiation of brittle fracture are always present in structures at sub-transition temperatures. This can easily be true under dynamic loading conditions, but can hardly apply statically unless there is some difference, not yet taken into account, between practical structures and the largest severely-notched laboratory specimens.

One such additional factor might be severe denting of a plate in the vicinity of an initial notch. The dent might easily create a field round the notch having a plastic stretch of the order of 1 per cent., equivalent to the original general yield situation, leaving a residual elastic stress field of the same extent with yield point magnitude. The conditions of initiation and the first stages of propagation would then be fulfilled, leaving the later stages of propagation to be cared for by the existence of the low, more uniformly distributed service stress. In this connection, a quantitative analysis is interesting. A surface energy of 4·5 ft. lb. per sq. in. demands an internal initial crack length of 0·72 in. for propagation to proceed in an elastic stress field of 20 tons per sq.in. level. When the propagating crack has reached a length of 11·5 in. the general stress level need only be 5 tons per sq.in. for the maintenance of propagation. If the crack is stopped at any stage, the process of initiation must be gone through again before further extension can take place.

Brittle Fracture at Butt Welds

A far more interesting and important situation is that encountered with fusion welding, where the cooling of the weld induces considerable plastic stretching, leaving the material in the vicinity in a state of residual stress. The simple case of a long butt weld between two wide plates is worthy of detailed investigation, because a good many service brittle failures can be traced to sources in such butt welds, the fracture eventually running transversely well into the plate on either side of the weld.

A recent American report[39] states, in relation to this, "Almost one-half of the serious tanker failures originated in butt welds of poor quality." This statement is remarkably confirmed by examination of Fig. 20, taken from a paper by Brown.[40] Here, the patterns of hull fractures in all the type T2 tankers have been superimposed on a single shell diagram. Many of the fractures are sufficiently short to make it obvious that they have originated at butt welds, and propagated into the plate perpendicularly to the lines of the welds. A similar diagram was published in 1945 by Adam[41] for fractures during shipbuilding (Fig. 21). This was stated to show a result of automatic arc welding in the open during cold weather. Fig. 22 shows a photograph of the origin of fracture in an oil storage tank in England. This fracture was also found to be associated with a weld probe which had been cut transverse to the weld, and in which a small portion of the weld had been removed for inspection, and the hole

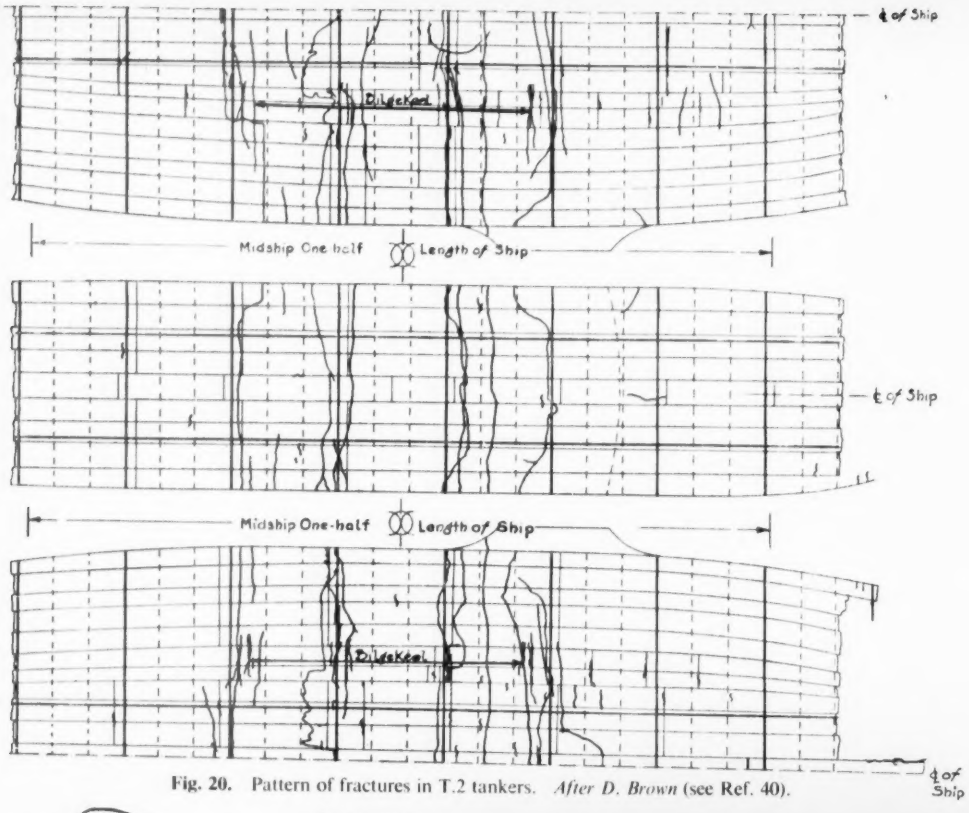


Fig. 20. Pattern of fractures in T.2 tankers. *After D. Brown* (see Ref. 40).

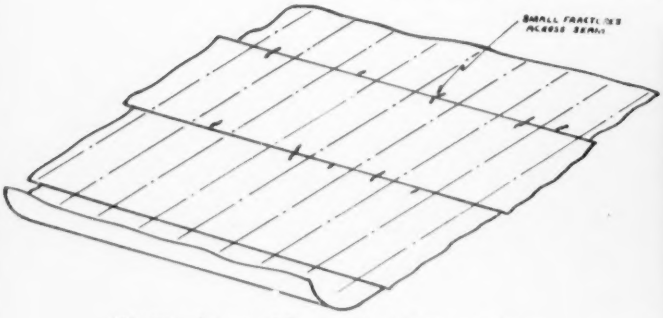


Fig. 21. Pattern of fractures in ship bottom plating. *After J. L. Adam* (see Ref. 41).

Fig. 23 (*below*). Typical short brittle fracture transverse to butt weld in oil storage tank in Europe: tank empty.

Fig. 22 (*above*). Source of complete brittle fracture in oil storage tank in Great Britain: tank partly filled.

later refilled with weld metal. On the same tank a short fracture of the type described above was also discovered and repaired before water testing. In Fig. 23 is shown one of a number of similar short fractures of the same type, which occurred after welding and before water testing in a similar oil storage tank in Europe. Long butt welds are not usual in bridge construction, but Fig. 24, taken from a paper by Graf,[42] illustrates the fracture which started, under similar conditions, at the weld between the web and flange of a large plate girder of low-alloy steel in the Hardenbergstrasse Bridge, Berlin. This fracture also propagated transversely into the plate.

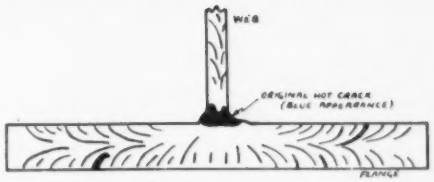


Fig. 24. Hardenbergstrasse Bridge fracture. Line drawing of photograph by O. Graf (see Ref. 42).

Theory of Residual Stresses

A study of the heat flow due to the welding operation is fundamental to this problem. Equations which can predict the temperature distribution due to moving sources of heat were first set up by Roberts,[43] and were applied to welding problems by Rosenthal.[44] The results have been simplified by the present author for practical application.[45] For the butt weld case, in which two-dimensional heat flow can be said to exist, the maximum temperature t above the surroundings, reached in the plate at distance x from the plane of motion of the heat source, is related to the source strength Q (cal. per sec. cm. of thickness) and velocity v, by the simple formula[45]—

$$t = \frac{Q}{8k\left(\frac{1}{5} + \frac{vx}{2\alpha}\right)} \quad .. \quad .. \quad .. (12)$$

where k and α are the thermal conductivity and thermal diffusivity of the material, respectively. α is related to the density ρ and specific heat c of the material by the relation $\alpha = k/\rho c$. Now, the parameter $\frac{vx}{2\alpha}$ is usually greater than unity outside the fused zone, except where the welding heat input per unit thickness is very small. It thus follows that, approximately—

$$t = \frac{Q}{4\rho cv} \times \frac{1}{x}, \frac{vx}{2\alpha} = \frac{1}{5} \quad .. \quad .. \quad .. (13)$$

It is interesting to note that equation (13) is almost identical with equation (4), for the instantaneous plane source, as for instance in a thermit butt weld, since Q/v is comparable with q. Equation (4) is evidently equivalent in form to equation (13), when v is large. The two equations differ in one respect, which is the relation between the total heat supplied and the size of the fused zone. If the width of this zone is $2b$ for a considerably large plate width $2a$, then the equalisation temperature T reached by all sections of the plate at some time after welding is complete, if all external heat losses are neglected, will be related to the melting temperature T_m, reached at the edge of the fused zone, by the relation—

$$\frac{T}{T_m} = \frac{b}{a} \quad .. \quad .. \quad .. \quad .. (14)$$

for conditions dictated by equation (4).

It has been shown[45] that with a moving source, more than twice the heat just necessary to melt the fused zone must be applied to allow for the two-dimensional heat conduction away from the weld pool. The ratio may be even greater than this for small heat inputs per unit thickness. For the conditions of equation (13), however, the equalisation temperature is given by—

$$\frac{T}{T_m} = \frac{2b}{a} \quad .. \quad .. \quad .. \quad .. (15)$$

In both these cases, however, the distribution of maximum temperature t with x can be determined from the simple inverse relation which follows from equations (4) and (13)

$$\frac{t}{T_m} = \frac{b}{x} \quad .. \quad .. \quad .. \quad .. (16)$$

There is an anomaly in this equation in comparison with equation (15), since $T > t$ when $x > \frac{a}{2}$. This is because equations (4) and (13) are related to infinite plates, but they are sufficiently accurate for practical purposes, assuming that t is only required to be accurate when $x < \frac{a}{2}$.

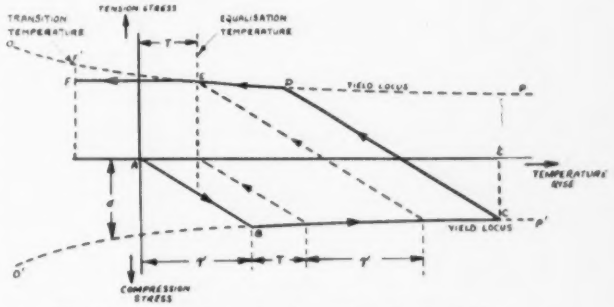


Fig. 25. Stresses due to heating and cooling in rigidly held bar.

With a knowledge of the temperature distribution during and after welding, and of the coefficient of expansion of the material β, the expansion history of the material adjacent to the weld may be traced. First, it is necessary to examine the behaviour of a bar, held rigidly at its ends, and heated and cooled. In Fig. 25 the change of stress is plotted against temperature. The yield loci OP, O′ P′, in tension and compression, are shown in Fig. 25. When heating is commenced, as shown by the line from point A, the bar sustains an elastic compressive stress proportional to temperature rise, until yield is first reached at B, when the temperature rise t has reached the value $\frac{\sigma}{E\beta} = \tau$, where E is Young's modulus, and σ is the yield stress relevant to that temperature. This temperature rise to reach the yield point is of the order of 150 deg. C. for steel. On further heating to temperature t, the stress remains at the relevant compressive yield (point C), but the bar sustains a total strain of $t\beta$, since free expansion is suppressed by the rigid ends. When the melting point is approached, this strain may well be of the order of 2 per cent. On cooling from t there is a short elastic phase when the stress changes uniformly with temperature drop until the tensile yield is reached at D. Thereafter the stress remains at the relevant tensile yield point until the point E is reached for temperature T.

Rodgers and Fetcher[46] seem to have been the first to assume that the longitudinal fibres in plates adjacent to a butt weld are held rigidly during the welding operation. The assumption is readily justified by the known fact that longitudinal distortion in butt welding seldom exceeds about 0·03 per cent. of the length. They presented a calculation of the residual stress system due to the making of a thermit butt weld 1·2 in. wide between the long edges of two mild

steel plates 30 in. by 15 in. by ¾ in. and compared this with measured stresses. Their calculation was carried out numerically, for each of thirty-seven time intervals after making the weld, using correct values of the physical constants relevant to each temperature. The approximate comparison between the calculated and measured residual stresses, shown in Fig. 26, does not seem to merit the painstaking method employed in calculation.

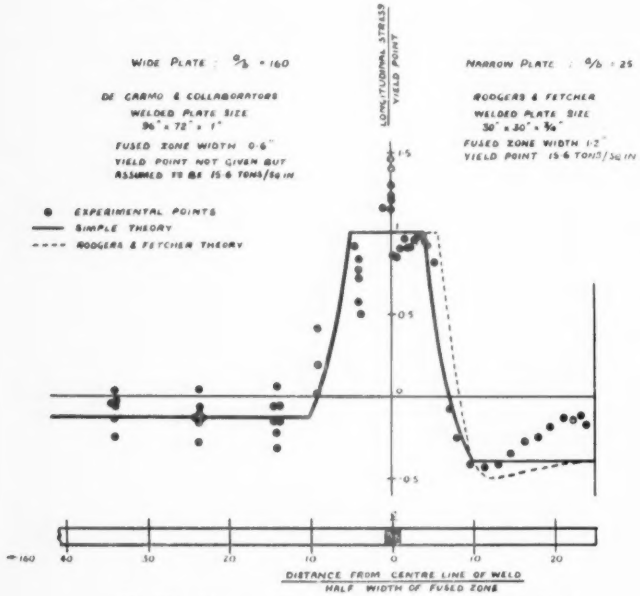


Fig. 26. Measured and calculated longitudinal residual stresses at butt welds.

A very simple alternative method of calculation is possible, using Fig. 25, and equations (16) and (14) or (15), which considers only the beginning and end temperature conditions. The final condition of the equalisation temperature is important because, when an isothermal condition is reached, additional uniform cooling is not accompanied by change of stress distribution. Further, as the final stresses seem to be determined mainly by temperatures only up to 350 deg. C., the consideration of changes of σ, β and E with temperature does not seem worthwhile.

When $T<\tau$, the length of the plate is assumed to be fixed, and changes of σ, β and E with temperature are neglected, it may be shown from Fig. 25 that the following residual stresses σ_r are created:

$$t>(T+2\tau),\ \sigma_r=\sigma$$
$$(T+2\tau)>t>\tau,\ \sigma_r=\sigma\left(\frac{t-T-\tau}{\tau}\right) \quad \ldots\ \ldots\ (17)$$
$$[t=(T+\tau),\ \sigma_r=0]$$
$$\tau>t, \qquad \sigma_r=-\sigma\frac{T}{\tau}$$

The condition of zero total longitudinal load in the plate is approximately satisfied by the above stress system, for the maximum temperature distribution of equation (16), when $\frac{T}{\tau}>\frac{1}{2}$.

The Rodgers and Fetcher case has been recalculated on this basis, using equations (14), (16) and (17), and $T_m = 1500$ deg. C., $T = 60$ deg. C., $\tau = 150$ deg. C., and the results are plotted in Fig. 26. Agreement with experiment is quite as good as that obtained by their original calculation.

A similar calculation, the results of which are also shown in Fig. 26, has been carried out for comparison with measurements due to De Garmo and collaborators[47] on 1 in. thick plates aggregating 96 in. by 72 in. when butt welded to give a fused width averaging 0·6 in. For this case T was taken to be 18·8 deg. C. since equation (15) was substituted for equation (14). The other treatment was as before. Again the agreement is quite good, when the experimental scatter is considered, although the measured σ_r is well above the yield point along the weld, and seems initially to fall off transversely somewhat more quickly than predicted.

Apart from the establishment of the simple physical basis of the butt weld longitudinal residual stress system, three points emerge from this experimental and theoretical analysis. Firstly, the heating and cooling cycles each impose on the material in the region of the fused zone successive plastic compression and tension strains of the order of 2 per cent. Secondly, the yield point residual tensile stress field extends away from the line of the weld to a total of three or more fused widths. Thirdly, the residual stress changes sign at about seven to nine fused widths, the greater figure relating to wider plates.

Residual Stresses and Brittle Fracture

It has been previously stated that a condition of severe plastic strain at the root of a notch is necessary for the initiation of brittle fracture with statically applied loads and with the material at a sub-transition temperature. In the notched specimens of prime plate, having no residual weld stresses, this was shown to be possible only when a general yield zone embraced the root of the notch. Now the extent of the plastic strain in these general yield zones, except as amplified by the presence of the notch itself, was frequently only of the order of 1 per cent. when fracture occurred. From examination of the slope of load/deformation curves taken for plain tensile specimens at elevated temperatures, there is no reason to doubt that comparable strain hardening can occur when the plastic strain is caused by cooling, just as much as when caused by extension at constant temperature. It is therefore reasonable to suppose that the conditions for brittle fracture initiation would be satisfied if a transverse crack of suitable length were present during the major part of a cooling cycle in the deposited metal at a butt weld, and the plate were later to be cooled to below the transition temperature. Transverse hot cracks of this type are sometimes found in weld metal. Usually they are confined within the fused zone, and do not penetrate the parent metal.

Greene[48] has shown that brittle fractures of this type may be initiated under laboratory conditions.* He welded together pairs of ¾ in. plates, 30 in. by 18 in. at their long edges, and produced transverse weld hot cracks at the centres of the plates by means of jewellers' saw cuts in the edges prepared for welding. One of these hot cracks spontaneously produced in the specimen a transverse brittle fracture 7 in. (or 16 fused widths) long when cooled to −30 deg. C. Six others, when not cooled to such a low temperature, gave similar fractures averaging 6·5 in. in length, after bending the specimens to create additional extreme-fibre bending stresses longitudinal to the welds, averaging 7·2 tons per sq.in., when based on plane bending theory. Similarly-treated specimens, when furnace stress-relieved at 650 deg. C., required considerably more than the fully plastic bending moment to be applied before fracturing, as would be expected of notched specimens cut from unwelded prime plate.

* See also Ref. 51.

It would appear from this that, although the initiation conditions may occasionally be satisfied by residual plastic strains and elastic stresses and sub-transition temperatures at a weld, slight additional loading is necessary in the majority of cases. The curve of Fig. 25 may perhaps explain this. It has been stated earlier that there is reason to believe that the transcrystalline strength of a mild steel is not unduly dependent upon temperature. It may well be, therefore, that the plastic stresses concentrated at the root of the notch have to be continuously maintained near their increasing top limit as cooling proceeds, in order to make the initiation positive. Now, after the equalisation temperature has been reached, after welding, no further change of residual strain or stress is induced by isothermal cooling. The cooling locus of Fig. 25 thus proceeds from E to F as cooling takes place. If the above hypothesis is correct, spontaneous brittle fracture from a transverse hot crack at a butt weld would be most likely to occur when the equalisation temperature itself was below the transition temperature of the material. This would explain spontaneous brittle fracture due to welding cold material. Otherwise, on cooling to the transition temperature along EF of Fig. 25, additional external load would be necessary to carry the locus from F to F^1 and so initiate the fracture. Similarly it becomes possible to understand the effect of preheating in raising the brittle strengths of notched welded plates, as has been shown to be the case by De Garmo.[50]

A remaining point, which has already been briefly commented upon, is the influence of surface energy in the residual stress type of brittle fracture. Simple calculations show that, with the lowest values of surface energy already measured, no hindrance to propagation is to be expected in the early stages, if the hot crack or other form of notch, extends to the full width of the fused zone. Even at this width the yield point elastic stress zone is quite wide enough to feed the necessary elastic energy to the propagating crack. The position for shorter cracks is not yet fully established. For instance, smaller surface energies than have yet been measured would be necessary if fully internal cracks, such as those treated by Sack (equation (11)), were to be propagated.

The role of surface energy seems important in deciding whether the brittle fracture at the butt weld shall travel for 6 in. or so, or cleave the structure completely. The tensile residual stress field has been shown, in general, to span no more than this distance of 6 in. on 1 in. plate, so that a service stress longitudinal to the weld of 9 tons per sq.in. would need to be superimposed to continue propagation at the surface energy level of 3·5 ft. lb. per sq.in.

Another case which might be explained in terms of surface energy, is the passage of a fairly long propagating brittle fracture through a riveted crack-arrester on a ship. This is known to have happened. Now, when a propagating fracture has reached a length of, say, 10 ft., the elastic energy available at a general stress level even as low as 5 tons per sq.in. would amount to 65 ft. lb. per sq.in. This might be quite sufficient to keep the crack going, even under the ductile conditions which sometimes accompany such a traverse of a riveted joint.

Quantitatively, these figures are impressive in their comparison with practical cases, although some of the fractures which have occurred would not have been possible unless the surface energies were a little lower than the minima yet determined. It would seem that better methods of evaluating minimum surface energy are still required if progress is to be maintained in the solution of the brittle fracture problem.

GENERAL CONCLUSIONS

The conclusions from this investigation may be stated under the three headings of main, subsidiary and practical. The main conclusions are those related directly to the observation of experimental facts. The subsidiary conclusions deal with experimental methods or working hypotheses. The practical conclusion is concerned with possible remedial action.

The main conclusions are as follows:

(1) When static external loads are applied to notched mild steel specimens having no residual stress or strain systems and showing usual ductility in the tensile test at temperatures within or just below the notch-brittle transition range (as determined by a static full-plate thickness test), brittle fracture from the root of a notch will not normally take place until general yield has occurred, and the general yield zone embraces the root of the notch allowing the notch to open. General yield is defined as that state in which no path exists wholly through elastic material between at least one pair of opposite external loading points. In the axially-loaded notched tensile specimen, general yield occurs when the net average stress across the notched zone reaches the yield point of the material for very thin plates, and two-and-a-half times this value when the plate thickness and depths of the notches are large compared with the notched width.

(2) Notch brittle fractures may be initiated in mild steels showing usual ductility in the tensile test, at temperatures within or just below the notch brittle transition range (as determined by a static, full-plate thickness test), when statically applied external loads are below those sufficient to cause general yield, if any extensive zone through the root of the notch has been subjected to large plastic flow previously to and independently from the external loading, so allowing the notch to open, and leaving a residual stress system of the order of yield point magnitude in the same zone. The primary example of this is the local thermal plastic strain system introduced in fusion welding along, and sometimes across, the line of the weld, by heating and cooling under restraint from adjacent cold metal.

(3) When the conditions for initiation of notch brittle fracture have been satisfied in the structure, either by shock, or by one of the static external loading conditions mentioned in conclusions (1) and (2) above, the elastic energy released in the body of the structure, as a result of change of length of the propagating fracture, reappears as plastic flow work along the fractured surfaces. In the case of ductile, or only partly brittle fractures, the plastic flow work may include energy contributions from external loads.

(4) A true notch brittle fracture in steel, after any form of initiation, cannot be propagated unless the rate of change with length of crack of the stored elastic energy in the specimen is equal to, or in excess of, a fundamental minimum fracture surface energy of the material at the given temperature. As far as is known, this minimum surface energy, which is not to be confused with the total energy absorption in any notch impact test, is of the order of 4·5 ft. lb. per sq.in. of the surface fractured for some mild steels at atmospheric temperatures. This propagation condition of minimum surface energy implies that for given temperatures and types of specimen there are critical initial crack lengths below which notch brittle fractures cannot take place without abnormally large external loading.

These four main observations are sufficient to explain the occurrence of notch brittle fractures, in full-scale structures such as described in the introduction, under static conditions, with small service loads.

As a result of the work previously described, certain other conclusions may be made. These are incidental to the main experimental facts, but are none the less important. They are as follows:

(*a*) The mechanism of initiation in notch brittle fracture under static load in mild steel showing usual ductility in the

tensile test, and at temperatures within or just below the transition range (as determined by a static full-plate thickness test) is identical with that for ductile fracture. By this mechanism the rapid increase of plastic strain at the root of the notch which follows general yield, or its equivalent, allows the stress there to concentrate, due to the resultant strain hardening and increased triaxiality, to a value reaching the limit which the material will withstand. This limit may be determined either by the cleavage or the shear strength of crystals in the aggregate, depending upon the temperature.

(*b*) The mechanisms of both initiation (under the influence of shock) and propagation of a notch brittle fracture of a mild steel showing usual ductility in the tensile test (as defined in conclusion (*a*) above), are the concentration of tensile stress at the root of the notch or crack to the limits of the material in shear and cleavage by the suppression of plastic flow due to very high strain rate. In the propagation of the partly brittle and partly ductile types of fracture, there is reason to believe that both the strain hardening and the high strain rate mechanisms exist together.

(*c*) Under the condition of propagation of notch brittle fracture in a mild steel showing usual ductility in the tensile test (as defined in conclusion (*a*) above), at a speed limited only by the velocity of stress waves, there is still fracture by mixed cleavage and shear along the surface of the crack, because of favourable, or unfavourable, orientation of the crystallographic shear and cleavage planes of individual crystals in the aggregate. It would also appear that the minimum surface energy of the material, at a given temperature, should be defined by the statistically fixed amount of shear of individual crystals which ought to take place at this high speed of propagation with random orientation.

(*d*) The minimum surface energy may be measured, for a given material and temperature, by means of a notched laboratory specimen pulled slowly in a testing machine. In order to secure the minimum value for the surface energy, the length of the notch for an internally notched specimen, or the net notched width for deep external notches, should be as small as is consistent with the retention of the brittle mode of failure under the given conditions. If the externally notched specimen is used, great care should be taken to establish truly axial loading. The surface energy may be measured conveniently either from the breaking load or by means of the recorded temperature-wave method, or by both methods together.

A practical conclusion from this work is that a certain prevalent type of notch brittle fracture, associated with transverse hot cracking of weld metal in long butt welds in commercial grades of mild steel over about $\frac{1}{2}$ in. in thickness, with moderate or no externally applied static load in the direction of the weld, depends for initiation and propagation on the residual systems of plastic strain and yield point elastic stress respectively, induced by the local heating and cooling during welding. The results of the work would also suggest that in conventionally designed welded mild steel structures, the chances of notch brittle fractures of this type occurring when only static external loads are to be withstood, at temperatures within or just below the transition range (as determined by a static notched bar full plate thickness test), should be considerably reduced if adequate residual stress relief were possible.

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The opinions expressed in the report are those of the author and not necessarily those of the British Welding Research Association.

APPENDIX I

The Determination of Surface Energy from the Temperature Wave: End Correction

If the plane instantaneous source of heat is bounded at $y = \pm a$, instead of $y = \pm\infty$, it may be shown that the temperature rise at any distance x from the plane of the source at $y = 0$, is given by[32]—

$$T = \frac{q}{4\pi_\rho c\alpha t} e^{\frac{-x^2}{4\alpha t}} \int_{-a}^{+a} e^{\frac{-y^2}{4\alpha t}} . \delta y = \frac{q\, e^{\frac{-x^2}{4\alpha t}}}{2_\rho c\sqrt{\pi\alpha t}} erf \frac{a}{\sqrt{4\alpha t}} \quad .. \quad (18)$$

where *erf* ( ) is the error function, tabulated in reference 32. This is the same as equation (3), with the addition of the *erf* term. The maximum temperature, determined by differentiation with respect to t, now occurs after the time t_o, given irrationally by—

$$\sqrt{\pi}\ erf\ z\ (m^2z^2 - \tfrac{1}{2}) - ze^{-z^2} = 0 \quad .. \quad .. \quad (19)$$

where $z = \frac{a}{\sqrt{4\alpha t_o}}$ and $x = ma$. Now, for practical purposes it is known that $z<1$. It is therefore possible to use the approximations—

$$\left.\begin{aligned} erf\ z &= \frac{2z}{\sqrt{\pi}} \\ e^{-z^2} &= 1 - z^2 \end{aligned}\right\} z<1 \quad .. \quad .. \quad .. \quad (20)$$

So that $$t_o = \frac{x^2}{2\alpha}\left(\tfrac{1}{2} + \frac{1}{4m^2}\right) \quad .. \quad .. \quad .. \quad (21)$$

The maximum temperature now becomes—

$$T_{max} = \frac{4q}{\pi_\rho cx} \cdot \frac{m}{(2m^2+1)^2},\ m>1 \quad .. \quad .. \quad .. \quad (22)$$

So that, in comparison with equation (4), the temperature correction factor to be applied in the above case, is given by—

$$\text{Correction factor} = \sqrt{\frac{\pi}{32e}} \cdot \frac{(2m^2+1)^2}{m},\ m>1 \quad .. \quad (23)$$

APPENDIX II

Fracture Propagation in a Transversely Notched Bar with Axial Tension

Deep Notches

Neuber[12] has shown that the elastic stresses may be calculated for a pair of deep external transverse notches in a bar under axial load, if an elliptic co-ordinate system is used. The boundaries of the notches are represented by the hyperbolae $v = v_o$. The relations between the cartesian (x, y) and elliptic (u, v) co-ordinates are then—

$$x = \sinh u \cos v,\ y = \cosh u \sin v \quad .. \quad .. \quad (24)$$

For axial load P (unit thickness) along the axis $y = 0$, and total width $2a$ between the roots of the notches, Neuber gives the stress function—

$$F = \frac{P}{2a} \cdot \frac{\sin v_o}{v_o + \sin v_o \cos v_o} (yv + \sin^2 v_o\ e^{-u} \cos v) \quad .. \quad (25)$$

The stresses at any point become:—

$$\sigma_u = \frac{1}{h}\frac{\delta}{\delta v}\left(\frac{1}{h}\frac{\delta F}{\delta v}\right) + \frac{1}{h^3}\frac{\delta h}{\delta u}\frac{\delta F}{\delta u}$$

$$\sigma_v = \frac{1}{h}\frac{\delta}{\delta u}\left(\frac{1}{h}\frac{\delta F}{\delta u}\right) + \frac{1}{h^3}\frac{\delta h}{\delta v}\cdot\frac{\delta F}{\delta v} \quad \ldots (26)$$

$$\tau_{uv} = \frac{-1}{h}\frac{\delta}{\delta v}\left(\frac{1}{h}\frac{\delta F}{\delta u}\right) + \frac{1}{h^3}\frac{\delta h}{\delta u}\frac{\delta F}{\delta v}$$

where

$$h = \sqrt{\left(\frac{\delta x}{\delta u}\right)^2 + \left(\frac{\delta y}{\delta u}\right)^2} \quad \ldots \ldots (27)$$

The boundary conditions at $v = v_o$ are $\sigma_v = \tau_{uv} = 0$. Along the axis $v = y = 0$, and when $v_o \to \frac{\pi}{2}$, as in an ideally sharp notch, it may then be shown that—

$$\sigma_u = \frac{P}{\pi a}\cdot\frac{1 + \sinh 2u}{\cosh^3 u}$$

$$\sigma_v = \frac{P}{\pi a}\cdot\frac{\cosh 2u - \sinh 2u}{\cosh^3 u} \quad \ldots \ldots (28)$$

$$\tau_{uv} = 0$$

A close approximation to the total strain energy W_T (unit thickness) contained within the area enclosed by co-ordinate u, is then given by the product of $\frac{P}{2}$ and the total extension along the axis $v = y = 0$, as calculated from equation (28).

$$W_T = \frac{P^2}{\pi E}\left[(1+\nu)(\tanh u + 2\log\cosh u) - 2\nu u\right] \quad \ldots (29)$$

E and ν are Young's modulus and Poisson's ratio, respectively.

The strain energy W_o contained within the same area defined by co-ordinate u if no notch is present is, simply

$$W_o = \frac{\pi P^2}{8E}\tanh u \quad \ldots \ldots \ldots (30)$$

the load P being assumed to be sustained over the total width $2a\cosh u$.

The excess energy due to the presence of the notch of total width $2a(\cosh u - 1)$ is then

$$W = \frac{P^2}{\pi E}\left[(1+\nu)(\tanh u + 2\log.\cosh u) - 2\nu u - \frac{\pi}{8}\tanh u\right] \quad (31)$$

The condition for propagation of the notch is

$$-\frac{\delta W}{\delta a} = 2S \quad \ldots \ldots \ldots (32)$$

where S is the surface energy of the material per unit length of crack. But

$$\frac{\delta W}{\delta a} = \frac{\delta W}{\delta u}\cdot\frac{\delta u}{\delta a}$$

Let $2a\cosh u = 2A = $ constant, then $a = A\,\mathrm{sech}\,u$.

$$\therefore -\frac{\delta W}{\delta a} = \frac{P^2}{\pi E}\cdot\frac{\left[(1+\nu)(\mathrm{sech}^2 u + 2\tanh u) - 2\nu - \frac{\pi}{8}\mathrm{sech}^2 u\right]}{A\,\mathrm{sech}\,u\tanh u}$$

Which reduces, when $u \to \infty$, to

$$\frac{\delta W}{\delta a} = \frac{P^2}{\pi E}\cdot\frac{2\cosh u}{A} = \frac{P^2}{\pi E}\cdot\frac{2}{a}$$

$$\therefore P = \sqrt{\pi E S a} \quad \ldots \ldots \ldots \ldots (33)$$

or $\sigma = \sqrt{\frac{\pi E S}{4a}}$, where σ is the critical mean stress across the minimum cross section.

This may be compared with the expression derived by Griffith[8] for an internal notch of total length $2a$:

$$\sigma = \sqrt{\frac{ES}{\pi a}}$$

Shallow Notch

For the calculation of elastic stresses in a bar with axial load and a single shallow notch, Neuber[12] uses the following co-ordinate system:

$$x = u + \frac{u}{u^2+v^2},\ y = v - \frac{v}{u^2+v^2} \quad \ldots \ldots \ldots (34)$$

When u and v are large, the net coincides with the cartesian net (x, y). The edge of the bar is given by $u = u_o$, and the notch is at $v = 0$, with depth $t = \frac{1}{u_o}$. When $u_o \to 1$, the notch is ideally sharp, with the external corners rounded off.

For axial stress σ at infinity, the stress function becomes

$$F = \frac{\sigma}{2}(u - u_o)^2\left[1 - \frac{1}{(2u_o^2 - 1)(u^2+v^2)}\right] \quad \ldots (35)$$

and the edge stress is given by

$$(\sigma_u)_{u_o} = \sigma\,\frac{1 - \frac{1}{(2u_o^2-1)(u_o^2+v^2)}}{1 + \frac{2v^2 - 2u_o^2 + 1}{(u_o^2+v^2)^2}} \quad \ldots \ldots (36)$$

Now, the axial extension along the edge is given by

$$U = \frac{1}{E}\int_o^y (\sigma_v)_{u_o}\delta y$$

$$= \frac{1}{E}\int_o^v (\sigma_v)_{u_o}\frac{\delta y}{\delta v}\cdot\delta v$$

$$= \frac{\sigma}{E}\int_o^v \left\{\frac{[(2u_o^2-1)(u_o^2+v^2)-1][(u_o^2+v^2)^2 - (u_o^2-v^2)]}{[2u_o^2-1]\,u_o^2+v^2\,(u_o^2+v^2)^2 - (2v^2-2u_o^2+1)}\right\}\delta v$$

$$= \frac{\sigma}{E_o}\left[v + \frac{2u_o^2}{2u_o^2-1}\left\{A\tan^{-1}\frac{v}{u_o+1} + B\tan^{-1}\frac{v}{u_o} + C\tan^{-1}\frac{v}{u_o-1}\right\}\right] \quad \ldots (37)$$

where

$$A = \frac{[u_o^2 + (u_o+1)^4]\,[u_o^2 - (u_o+1)^4 - 2] + 1}{2[u_o+1]^3\,[3(u_o^2+(u_o+1)^4)^2 + 4(u_o+1)^4 + 1]}$$

$$B = \frac{[u_o^2 + u_o^4]\,[u_o^2 - u_o^4 - 2] + 1}{2u_o^3\,[3(u_o^2+u_o^4)^2 + 4u_o^4 + 1]}$$

$$C = \frac{[u_o^2 + (u_o-1)^4]\,[u_o^2 - (u_o-1)^4 - 2] + 1}{2[u_o-1]^3\,[3(u_o^2+(u_o-1)^4)^2 + 4(u_o-1)^4 + 1]}$$

when $u_o \to 1$, $A = \frac{9}{466}$, $B = \frac{3}{34}$, $C = 0$, $t = 1$.

The excess axial extension along an edge containing a sharp notch of depth t, small in comparison with the total width of bar $2a$, is therefore given by

$$U_e = \frac{2\sigma t}{E}\left(\frac{9}{233} + \frac{3}{17}\right)\frac{\pi}{2} \quad \ldots \ldots \ldots (38)$$

The average extension of the bar will be one-half of this, if plane sections remain plane, remote from the notch. The excess elastic energy is then

$$W = \frac{\pi\sigma^2 a t}{E} \times 0{\cdot}2151 \quad \ldots \ldots (39)$$

For the propagation of the crack

$$-\frac{\delta W}{\delta t} = S$$

where S is the surface energy of the material.

$$\therefore \sigma = \sqrt{\frac{ES}{0{\cdot}2151\pi a}} \quad \ldots \ldots (40)$$

or, approximately

$$\sigma = \sqrt{\frac{3ES}{2a}}$$

and will be the same whether the bar is notched on one or both sides.

Surface Energy Formulae in Engineering Units

For convenient use, in opposition to the acceptance of the more logical definitions used in derivation, it is suitable to define the operative lengths in the surface energy formulae as *l*. *l* is thus the complete length or diameter of an internal crack, or the notched ligament width of an externally notched bar.

Similarly, the failing stress σ is now defined for convenience as the mean stress across the notched section, whether externally or internally notched. This is only strictly true for the deeply externally notched case but, as the remaining three formulae apply only when the notch length is small compared with the total width, the error is not large.

The four formulae which are available now become, in tons per sq.in., inch and ft. lbs. per sq.in. units, using E = 13,500 tons per sq.in. and $\nu = 0{\cdot}3$:

(1) Plane internal notch (Inglis-Griffith)

$$\sigma = 6{\cdot}8\sqrt{\frac{S}{l}},\ S = \frac{\sigma^2 l}{46{\cdot}3},\ l = \frac{46{\cdot}3S}{\sigma^2}$$

(2) Circular internal notch (Neuber-Sack)

$$\sigma = 11{\cdot}2\sqrt{\frac{S}{l}},\ S = \frac{\sigma^2 l}{125},\ l = \frac{125S}{\sigma^2}$$

(3) Plane deep external notches (Neuber-Wells)

$$\sigma = 10{\cdot}7\sqrt{\frac{S}{l}},\ S = \frac{\sigma^2 l}{114},\ l = \frac{114S}{\sigma^2}$$

(4) Plane shallow external notch (Neuber-Wells)

$$\sigma = 14{\cdot}7\sqrt{\frac{S}{l}},\ S = \frac{\sigma^2 l}{216},\ l = \frac{216S}{\sigma^2}$$

. . (41)

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